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Tacit Collusion in Capacity Investment: The Role of Capacity Exchanges

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Abstract. In many capacity-intensive industries (e.g. electricity, bandwidth), exchanges allow firms to buy and sell wholesale capacity before selling on the retail market. This allows firms to smooth demand shocks, but it also raises suspicions that exchanges facilitate tacit collusion to limit capacity investment. This paper models investment and exchange in a one-shot game and in a repeated game with tacit collusion. It finds that the presence of the exchange does not reduce total capacity investment, and thus does not raise consumer prices. In fact, the exchange may make it *more* difficult to sustain tacit collusion.

Keywords: capacity investment; capacity exchanges; business to business exchanges; tacit collusion.

Tacit Collusion in Capacity Investment:
The Role of Capacity Exchanges

1 Introduction

In many industries where capacity constraints are important, such as energy and telecommunications, firms can trade capacity in an exchange. These exchanges are modeled on commodities exchanges for products like oil and soybeans, but they are focused on trades between potentially competing firms.[1] This paper examines how capacity exchanges affect the potential for an oligopolistic industry to tacitly collude on the initial capacity investment.

It is important to distinguish between intermediate good exchanges, which are two-sided markets that connect customers with suppliers, and capacity exchanges, which are essentially one-sided markets where firms trade with one another. As an example, Kwoka (2001) studied exchanges in the automobile industry and noted that "The competitive concerns over B2B exchanges fall into two broad categories – those involving the final output of the exchange participants (for example, cars) and those involving the products transacted on the exchanges themselves (e.g., wiring or tires)" (pg. 66).

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[1]Some trades have been misused in accounting frauds (e.g. ENRON and Global Crossing), but the legitimate benefits are sufficient that exchanges will continue to be important.

In commodity infrastructure markets like electricity or telecommunications bandwidth, these are close to one and the same.

Lucking-Reiley and Spulber (2001) surveyed business-to-business exchanges of both types and note that they can enhance efficiency but also raise antitrust concerns, in particular the possibility of tacit collusion. The concern is echoed by Kühn (2001) who proposes that exchanges should only make available aggregate data (as indeed they do in our model).

There are two distinct routes by which the introduction of a capacity exchange can affect tacit collusion. The first is that the exchange may alter the information structure of the game firms play. Imperfect information about other firms' actions means that cooperative actions could be mistaken for defections, and price wars could result (Green and Porter 1982). A capacity exchange might be one of a number of ways to improve information and thus facilitate tacit collusion (Kühn 2001). Similarly, an exchange could be used as a means of punishment in the event of defection. For example, a simple punishment strategy would just bar a defecting firm from using the exchange.

The other route by which a capacity exchange could affect tacit collusion is uncertainty over the demand for capacity. Uncertainty implies that investment can be *ex post* inefficient, and a capacity exchange will therefore

alter the payoff structure of the game – in particular it alters the defection payoff in a tacit collusion arrangement. This effect is the focus of this paper.

We consider industries where a small number of firms make long-lived investments in capacity. Firms thus choose quantities à la Cournot, and as in the standard Cournot model the entire quantity is sold at the market-clearing price. After the capacity commitments are made, there are random shocks to retail demand; thus, firms have too much or too little capacity relative to the *ex post* Cournot optimum.

In this environment, a capacity exchange is only desirable if the firms have differentiated retail demand curves and at least partially uncorrelated demand shocks. Otherwise, the total quantity would simply be sold at the price the market would bear, and no amount of capacity trading would change that price. Thus, we model goods which are perfect substitutes from the firms' point of view, but imperfect substitutes from the point of view of retail consumers. For example, in markets like telecommunications and electricity, bandwidth and megawatts are homogeneous in the wholesale market, but they are bundled with associated services and sold in multiple, overlapping retail markets subject to diverse demand shocks.

With a capacity exchange, firms with more demand can buy extra capacity from those with less demand. This allows all firms to sell an amount that

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is closer to their optimal quantity in the retail market. Under the Cournot assumption that retail prices adjust to clear the market, this effect raises profits but not the expected price, so social surplus unambiguously rises.[2] In a repeated game, the firms can tacitly collude to reduce total capacity even without an exchange. Adding the exchange actually increases the incentive to defect, so that it is more difficult to sustain tacit collusion with the exchange than without it. We believe this is a novel result that needs to be weighed against the information effects mentioned above.[3]

The paper is organized as follows. Section 2 presents the basic model. In section 3, we apply that model in a static setting without an exchange and then add the exchange in section 4. In section 5 we move to the repeated game setting, and we conclude in section 6.

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[2]If demand shocks were correlated across firms (e.g. a general recession or widespread hot weather), all firms would have either excess demand or supply, and trading could not improve the outcome.

[3]Somewhat comparable results by Maksimovic (1988) and Stenbacka (1994) show that firms with higher debt-equity ratios have more difficulty sustaining tacit collusion and that firms may choose less debt to avoid this problem. In their setting, debt raises the defection payoff just as the exchange does in this paper.

2 Model

There are N firms, indexed by $j = 1, \ldots, N$, each producing a single good. Retail demand for each firm's good is linear:

$$p_j = \alpha - \beta q_j - \gamma \sum_{i=1}^{N} q_i + \epsilon_j \tag{1}$$

where p_j is the retail price per unit and q_j is the quantity made available by firm j. Parameters β and γ indicate that retail price responds to a firm's own quantity and to the total quantity in the market. Thus $\beta = 0$ represents the conventional linear Cournot setup, while $\gamma = 0$ represents monopoly. Demand also includes a random shock, ϵ_j, with expected value 0 and variance σ^2. The shocks are firm-specific and are independent and identically distributed.[4]

Firms play a three-stage game:

Stage 1: Capacity Investment Each firm chooses to build capacity k_j. Let all firms have identical constant marginal cost of capacity c.

Stage 2: Capacity Trading Each firm learns the realization of its ϵ_j and then chooses a quantity δ_j that it will trade in the exchange. If $\delta_j < 0$, firm j sells capacity into the exchange; if $\delta_j > 0$, firm j buys additional capacity. The price at which capacity is bought and sold is

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[4]We assume that the support of ϵ_j is sufficiently bounded to prevent negative prices.

determined in the exchange and is denoted s. Firms are price-takers in the exchange.[5]

Stage 3: Retail Sales In the third stage, the quantity that firm j sells to retail consumers is $q_j = k_j + \delta_j$ with p_j given by (1).[6]

The next three sections analyze the game under increasingly complex conditions. We first examine a benchmark with no exchange. Next, we consider firms that trade in an exchange in one period only. Finally, we extend the model to a repeated game.

3 Competition Without a Capacity Exchange

Consider a benchmark case in which there is no exchange, i.e. $\delta_j = 0 \; \forall j$. We solve the game backwards to find a subgame perfect equilibrium. By stage 3, each firm has chosen its k_j. The profit of firm j is

$$\Pi_j = \left(\alpha - \beta k_j - \gamma \sum_{i=1}^{N} k_i + \epsilon_j \right) k_j - c k_j \qquad (2)$$

There is no stage 2, since there is no exchange. In stage 1, firm j chooses k_j. However, the value of ϵ_j has not yet been realized, so firm j must

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[5]We discuss relaxing this assumption in Section 6.

[6]There is no strategic decision in "stage" 3, so it is not properly a subgame, but it is easier to describe the model if we separate retail sales from the wholesale trading market. Thus we slightly abuse game theory terminology and call this a three-stage game.

maximize the expected value of Π_j. Solving simultaneously for all firms gives the Cournot equilibrium quantity:

$$k^{NT} = \frac{\alpha - c}{2\beta + \gamma + \gamma N}$$

where the superscript NT denotes "No Trading." The firm's expected profit based on this optimal investment is

$$E(\Pi^{NT}) = (\alpha - (\beta + \gamma N)k^{NT})k^{NT} - ck^{NT} \tag{3}$$

These are just the familiar Cournot results using the β and γ notation.

4 Competition With a Capacity Exchange

Now introduce trading into the one-shot game. Since firms have symmetric costs, the only incentive to trade is to smooth demand shocks.

4.1 Stage 3 Retail Sales

In stage 3, both k_j and δ_j are given. The quantity that firm j sells to retail consumers is $k_j + \delta_j$ and the price it charges is

$$p_j = \alpha - \beta(k_j + \delta_j) - \gamma \sum_{i=1}^{N}(k_i + \delta_i) + \epsilon_j \tag{4}$$

The firm's retail revenue is $R_j = p_j(k_i + \delta_i)$

4.2 Stage 2 Capacity Trading

By stage 2, capacity k_j is set, but firm j can choose δ_j, the amount of capacity that it buys or sells. The firm maximizes its operating profit from stage 3 retail sales minus trading costs:

$$\max_{\delta_j} \pi_j = R_j - s\delta_j \tag{5}$$

where s is the price of capacity in the capacity exchange. We assume that the firms all behave as price takers with regard to trading. That is, they do not expect that their trades will influence s.[7]

Since no new capacity can be built during stage 2, $\sum_{i=1}^{N} k_i$ remains unchanged despite the trading. The first order condition for (5) is firm j's *trading curve* as a function of s:

$$\delta_j(s) = \frac{\alpha - \gamma \sum_{i=1}^{N} k_i + \epsilon_j - s}{2\beta} - k_j \tag{6}$$

The total amount of capacity sold in the trading market must equal the total amount purchased. The market clears if $\sum_{i=1}^{N} \delta_i = 0$. Combining the market clearing condition with the trading curves of each firm, the equilibrium price is:

$$s^* = \alpha - (2\beta + \gamma N)\frac{\sum_{i=1}^{N} k_i}{N} + \bar{\epsilon}$$

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[7]See the discussion in Section 6 on relaxing this assumption.

where $\bar{\epsilon} = \sum_{i=1}^{N} \epsilon_i$.

Finding the equilibrium quantities in the trading market is difficult, primarily because of very tedious algebra. The following proposition gives results that apply to this model and a range of similar linear-demand oligopoly models:

Proposition 1 *Let firm j have trading curve $\delta_j(s) = A_j + B_j\epsilon_j + D_j s - k_j$ and let this firm sell its output at retail price $p_j = F_j + G_j\epsilon_j + H_j(k_j + \delta_j) + L_j \sum_{i=1}^{N} k_i$. Let demand shocks $\epsilon_1, ..., \epsilon_N$ be i.i.d. random variables with mean 0 and variance σ^2. If all firms behave as price takers in the capacity exchange, then the quantity traded by firm j is*

$$\delta_j(s^*) = \hat{A}_j + B_j\epsilon_j - D_j \sum_{i=1}^{N} \hat{D}_i\epsilon_i - k_j$$

and the expected operating profit of firm j (net of trading costs) is

$$E(\pi_j) = \tilde{A}_j + \tilde{B}_j\sigma^2$$

where the coefficients $\hat{A}_j, \tilde{A}_j$, etc. are defined in the proof and are functions of the capacities, k_i, and the parameters only.

The proof of Proposition 1 and all subsequent propositions are given in the Appendix. Applying the proposition to the Cournot case of (4) and (6)

gives equilibrium capacity trade for firm j:

$$\delta_j(s^*) = \overline{k} - k_j + \frac{\epsilon_j - \overline{\epsilon}}{2\beta}$$

where $\overline{k} = \frac{\sum_{i=1}^{N} k_i}{N}$. Firms with below-average capacity or above-average demand will buy more in the capacity exchange. Under trading, a firm's retail quantity supplied and retail price charged depend on its own physical capacity only insofar as its own capacity affects the total amount of capacity installed by all firms. This shows how capacity trading smooths outcomes relative to the no-trading case.

The expected operating profit of firm j is

$$E(\pi_j) = (\alpha - (\beta + \gamma N)\overline{k})k_j + \beta(\overline{k} - k_j)\overline{k} + \frac{N-1}{4\beta N}\sigma^2 \qquad (7)$$

The intuition behind (7) is that trading has three effects. First, the firm trades, on average, to the industry average capacity, so a firm's own retail price is determined entirely by the industry average rather than its own quantity. Second, there is a benefit or cost to having capacity different from the industry average (but this effect disappears in the symmetric equilibrium we will discuss below). Third, there is a constant benefit from smoothing the random shocks which is proportional to the variance of the shocks.

4.3 Stage 1 Capacity Investment

In Stage 1, firms anticipate the capacity trading equilibrium of stage 2, and therefore they anticipate operating profits $E(\pi_j)$. Each firm maximizes net profit $E(\Pi_j) = E(\pi_j) - ck_j$ by choosing k_j. We employ the usual Cournot assumption that each firm takes all of the other firms' capacities as fixed. The first order condition is

$$\frac{dE(\Pi_j)}{dk_j} = \alpha - (2\beta + \gamma N)\left(\overline{k} + \frac{k_j}{N}\right) + \frac{2\beta\overline{k}}{N} = c$$

In equilibrium, all of the firms solve this condition. Solving simultaneously, the equilibrium capacity choice is:

$$k^T = \frac{\alpha - c}{2\beta + \gamma + \gamma N} = k^{NT}$$

where superscript "T" stands for trading. Trading does not change the level of capacity investment from the no-trading case. Substituting this level of capacity investment into the expected profit gives:

$$E(\Pi_j^T) = E(\Pi_j^{NT}) + \frac{N-1}{4\beta N}\sigma^2 \tag{8}$$

Capacity trading allows firms to handle demand shocks better, thus increasing profits. The gains are greater when the variance of the random shock is higher. Since $\frac{N-1}{N}$ is concave, most of the gains from capacity trading come with just a few firms participating.

The size of the increased profits does not depend on how vigorous down-stream competition is (i.e. how large γ is). In this model, trading is not a mechanism to increase market power or hurt the consumer; it is purely a way for firms to increase profits by smoothing shocks. Consumers neither gain nor lose on average, since total capacity installed is unchanged.

5   Repeated Games With a Capacity Exchange

The results thus far hold in a conventional one-shot model. We now examine whether the exchange will be less efficient in a repeated game setting.

Tacit collusion with trading potentially involves two effects: coordination on stage 1 capacities and coordination on trading in the exchange in stage 2. Strategies to punish defection also involve two potential effects: in the event of defection, firms could alter stage 1 capacities or alter their stage 2 trading quantitities, or both.

To narrow this large set of problems, we focus on cooridnation and pun-ishments involving stage 1 capacities. Exchange behavior continues to be price-taking (and therefore efficient), so colluding firms have no collective incentive to alter exchange behavior (because no improvement is possible).

The entire game, stages 1-3, is repeated infinitely and all actions of all firms are perfectly observable. The simplest form of tacit collusion relies on

a trigger strategy equilibrium (Friedman 1971). All firms install a collusive capacity in stage 1, then trade as in Section 4 in stage 2, and sell to customers in stage 3. If any firm tries to cheat on this equilibrium path, then all firms revert to Nash behavior forever.

Martin (2002) presents a nice textbook treatment of this type of repeated game and shows that cooperation is sustainable as long as the period-to-period interest rate r is not too large:

$$\frac{1}{r} \geq \frac{\text{payoff to defecting} - \text{payoff to colluding}}{\text{payoff to colluding} - \text{Nash payoff}} \tag{9}$$

where each of the payoffs occurs in just one repetition of the game. We now find these payoffs for our model.

Section 4 discussed how the Nash payoffs are similar in the trading and no-trading cases but for the addition of a constant (equation (8)). The collusive payoffs are also similar. In the no-trading case, the firms coordinate and choose k_{COL}^{NT} to solve:

$$\max_{k} E(\Pi^{NT}) = (\alpha - \beta k - \gamma N k) \, k - ck$$

The solution is $k_{COL}^{NT} = \frac{\alpha - c}{2\beta + 2\gamma N}$, with a corresponding expected profit $E(\Pi_{COL}^{NT})$.

With trading, firms jointly maximize expected profits given by (7). Since all firms are identical before the demand shocks are realized, the collusive quantity is the same for each firm. Rewriting (7) with $\overline{k} = k$ gives the

collusive maximization problem with trading:

$$\max_k(\alpha k - \beta k + \gamma N k)k + \frac{N-1}{4\beta N}\sigma^2 - ck$$

By inspection, this is the same as the no-trading problem except for the addition of a constant; hence the collusive quantity is the same with or without the exchange. Thus, we denote the collusive quantity by k_{COL} either with or without trading, and we find that the difference between the collusive profits with and without trading is just a constant:

$$E(\Pi_{COL}^T) = E(\Pi_{COL}^{NT}) + \frac{N-1}{4\beta N}\sigma^2$$

Since both the collusive and Nash payoffs only differ by a constant which cancels in the denominator of (9), the change in interest rates that support tacit collusion is determined entirely by the defection payoff in the numerator. Comparing the defection capacities under the two regimes leads to a surprising result:

Proposition 2 *The defection capacity under trading is greater than the defection capacity without trading: $k_{DEF}^T > k_{DEF}^{NT}$.*

The intuition behind this result is that the payoffs to defection are different under the different regimes. Without trading, firm j maximizes (3) for the case where it chooses k_j while all other firms choose k_{COL}. First define $\overline{k}_{DEF} = \frac{k_j + (N-1)k_{COL}}{N}$ as the industry average capacity when all firms

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except firm j choose the collusive capacity. Then k_{DEF}^{NT} solves:

$$\max_{k_j}(\alpha - (\beta + \gamma N)\overline{k}_{DEF})k_j + \beta(\overline{k}_{DEF} - k_j)k_j - ck_j$$

Defection under trading is more complicated because it involves two effects. First, as in any collusion model, the defecting firm gains market share while suffering only a small decline in price. Second, the defecting firm has more capacity than the others, so it tends to sell capacity in the exchange. These sales are profitable and augment the incentive for a defecting firm to increase capacity. Rewriting (7), the trading defection capacity k_{DEF}^{T} solves:

$$\max_{k_j}(\alpha - (\beta + \gamma N)\overline{k}_{DEF})k_j + \beta(\overline{k}_{DEF} - k_j)\overline{k}_{DEF} + \frac{N-1}{4\beta N}\sigma^2 - ck_j$$

The trading maximization problem is different because trading allows the defecting firm to gain market share but mute the price-reducing effect on its own price. It does this by selling some of the extra capacity to other firms, which spreads the price-reduction to each of the firms. In essence, trading allows the defector to gain all the benefits of cheating but force some of the costs onto the other firms.

Mathematically, the difference is in the second terms of the objective functions. Without trading, the defecting firm suffers the full effect of its defection capacity, but with trading, it suffers only to the extent that it raises the industry average capacity. Thus, defection profits must be higher

under trading than they are without trading, even net of the constant term. This implies:

Proposition 3 *Tacit collusion can be sustained at higher interest rates without trading than with trading.*

6   Conclusion

Capacity exchanges have an obvious benefit since they allow firms to smooth firm-specific demand shocks. At the same time, they may give the appearance of fostering collusion among competing firms. We showed that in a static game, the exchange does raise firm profits and does not benefit consumers on average, but neither does it increase market power. When this game is repeated, we showed that the exchange actually has a pro-competitive effect in the sense that it may disrupt tacit collusion. Thus, there is a countervailing force in capacity exchanges that works against any tacit collusion that may come from information and communication.

It is important to recognize that firms may not behave as perfect competitors in capacity exchanges. In this case, our efficiency results would be diluted. Strategic bidding would make the exchange less efficient and lower the firms' profits. But we believe that strategic bidding in the exchange cannot increase average prices in the retail market as long as all capacity

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must be sold eventually.

There are many possible extensions to the tacit collusion model. For example, the punishment for defecting could come entirely through the exchange itself rather than changes in physical capacity. Also, not all variables may be perfectly observable, in which case the exchange could facilitate collusion by providing information. Then the pro-competitive efficiency effect in this paper would have to be traded off against the anti-competitive information effect.

Appendix

Proof of Proposition 1: The price in the exchange is determined by the equilibrium condition (all summations are from $i = 1$ to N):

$$\sum \delta_i(s^*) = 0$$

$$\sum A_i + \sum B_i \epsilon_i + \sum D_i s^* = \sum k_i$$

$$s^* = \frac{\sum A_i + \sum B_i \epsilon_i - \sum k_i}{-\sum D_i}$$

Substituting s^* into $\delta(s)$, the quantity traded by firm j is:

$$\left(A_j - D_j \frac{\sum A_i}{\sum D_i} + \frac{D_j}{\sum D_i} \sum k_i \right) + B_j \epsilon_j + D_j \frac{-\sum B_i \epsilon_i}{\sum D_i} - k_j$$

$$= \hat{A}_j + B_j \epsilon_j + D_j \sum \hat{D}_i \epsilon_i - k_j$$

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This gives the trading curve. The firm's capacity after trading is

$$q_j^* = \delta_j(s^*) + k_j$$

Given these trades, the retail price is:

$$
\begin{aligned}
p_j^* &= F_j + G_j \epsilon_j + H_j q_j^* + L_j \sum_{i=1}^{N} k_i \\
&= \left(F_j + H_j \hat{A}_j + L_j \sum_{i=1}^{N} k_i \right) + (G_j + H_j B_j)\epsilon_j + H_j D_j \sum_{i=1}^{N} \hat{D}_i \epsilon_i \\
&= \hat{F}_j + \hat{G}_j \epsilon_j + H_j D_j \sum_{i=1}^{N} \hat{D}_i \epsilon_i
\end{aligned}
$$

Then the operating profit, net of the trading costs, is found by several steps of algebra which lead to:

$$
\begin{aligned}
\pi_j &= p_j^* q_j^* - s^*(q_j^* - k_j) \\
&= \hat{A}_j \hat{F}_j - (\hat{A}_j - k_j)\left(\frac{\hat{A}_j - A_j}{D_j} \right) \\
&\quad + \left(\hat{A}_j \hat{G}_j + B_j \hat{F}_j - B_j \left(\frac{\hat{A}_j - A_j}{D_j} \right) \right) \epsilon_j \\
&\quad + (B_j \hat{G}_j)\epsilon_j^2 \\
&\quad + \left(\hat{A}_j D_j H_j + D_j \hat{F}_j - D_j \left(\frac{\hat{A}_j - A_j}{D_j} \right) - \hat{A}_j + k_j \right) \sum \hat{D}_i \epsilon_i \\
&\quad + (B_j D_j H_j + D_j \hat{G}_j - B_j)\epsilon_j \sum \hat{D}_i \epsilon_i \\
&\quad + (D_j^2 H_j - D_j) \left(\sum \hat{D}_i \epsilon_i \right)^2
\end{aligned}
$$

When we take the expected value of this operating profit, we can use the i.i.d. assumption to simplify the above. In particular, we have $E(\epsilon_j) = 0$,

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$E(\epsilon_i \epsilon_j) = 0 \forall i \neq j$, and $E(\epsilon_j^2) = \sigma^2$. Then:

$$
\begin{aligned}
E(\pi_j) &= \hat{A}_j \hat{F}_j - (\hat{A}_j - k_j)\left(\frac{\hat{A}_j - A_j}{D_j}\right) \\
&\quad + (B_j \hat{G}_j)\sigma^2 \\
&\quad + (B_j D_j H_j + D_j \hat{G}_j - B_j)\hat{D}_j \sigma^2 \\
&\quad + (D_j^2 H_j - D_j)\sum \hat{D}_i^2 \sigma^2
\end{aligned}
$$

This is equal to a constant plus a collection of terms multiplied by the variance, hence we can write it

$$
E(\pi_j) = \tilde{A}_j + \tilde{B}_j \sigma^2
$$

Proof of Proposition 2: We want to show that:

$$
k_{DEF}^T > k_{DEF}^{NT}
$$

$$
\frac{\alpha N - cN - (2\beta - 2\beta N^{-1} + \gamma N)(N-1)k_{COL}}{4\beta - 2\beta N^{-1} + 2\gamma N} > \frac{\alpha - c - \gamma(N-1)k_{COL}}{2\beta + 2\gamma}
$$

$$
\frac{1}{k_{COL}}\frac{\alpha N - cN - (2\beta - 2\beta N^{-1} + \gamma N)(N-1)k_{COL}}{2\beta - \beta N^{-1} + \gamma N} > \frac{1}{k_{COL}}\frac{\alpha - c - \gamma(N-1)k_{COL}}{\beta + \gamma}
$$

$$
\frac{\gamma N^2 + 4\beta - 2\beta N^{-1} + \gamma N}{2\beta - \beta N^{-1} + \gamma N} > \frac{2\beta + \gamma N + \gamma}{\beta + \gamma}
$$

$$
\frac{\gamma N^2 + 2\beta - \beta N^{-1}}{2\beta N - \beta + \gamma N^2} > \frac{\beta N^{-1} + \gamma}{\beta + \gamma}
$$

$$
\frac{\gamma \beta N^2 + 2\beta^2 + 3\beta\gamma - 2\beta\gamma N}{(2\beta N - \beta + \gamma N^2)(\beta + \gamma)} > \frac{\beta N^{-1}(\gamma N^2 + 2\beta N + \gamma)}{(2\beta N - \beta + \gamma N^2)(\beta + \gamma)}
$$

$$
\gamma N^2 + 3\gamma - 3\gamma N - \gamma N^{-1} > 0
$$

$$
N(N-3) + (3 - N^{-1}) > 0
$$

This holds for any $N > 1$, proving the proposition.

Proof of Proposition 3: We want to show that the defection payoff is higher under trading. Suppose firm j played k_{DEF}^{NT} in the trading case. Since

$$k_{DEF}^{NT} > \bar{k}_{DEF}^{NT} = \frac{k_{DEF}^{NT} + (N-1)k_{COL}}{N}$$

the second term in the profit function would be greater in the trading case than the non-trading case. So playing the suboptimal strategy k_{DEF}^{NT} would yield higher profits under trading. The optimal strategy k_{DEF}^{T} must yield even higher profits.

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