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The Nash Bargaining Solution in Labor Market Analysis[1]

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Abstract

The non-symmetric Nash bargaining solution is frequently applied in the study of labor market outcomes, but the axiomatic approach in which it is grounded offers little guidance as to the determinants of agents' threat points and relative bargaining power. This paper modifies the Rubinstein-Wolinsky (1985) sequential matching and bargaining model to study the role of individual bargaining costs, status quo payoffs, and outside options in determining bargaining power weights and threat points in Nash bargaining solution. Key results differentiate the strategic implications of fixed and time discount-based bargaining costs and demonstrate the general validity of the Nash bargaining solution in characterizing steady-state market outcomes in which outside options are endogenously determined. In this scenario, agents' relative bargaining weights depend on their matching probabilities.

Keywords: Nash bargaining solution, strategic bargaining, outside options, status quo payoffs, labor markets, matching and bargaining

JEL Codes: C78, J31, J52

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1. Introduction

The Nash bargaining solution (Nash, 1950, 1953) has long been the workhorse model for analyzing bilateral labor market transactions in which both parties are understood to enjoy some power to influence the terms of exchange.[2] This is presumptively the case in employment relationships governed by collective bargaining agreements, but recent surveys of U.S. workers (Hall and Krueger 2010, 2012) and German employers (Brenzel *et al*. 2014) indicate that wage bargaining frequently occurs in non-union hires as well, especially for higher-skill workers and in labor markets with lower unemployment rates. Of particular relevance for the analysis of bargaining between firms and workers is the *nonsymmetric* version of Nash's model, which allows for differences in bargaining power between agents (Kalai 1977).

This bargaining solution is appealing for its simplicity and analytical tractability. Based on an austere set of elements characterizing a bargaining relationship and a small number of plausible axioms, Nash's approach yields clear and broadly intuitive implications regarding the role of these elements in determining bargaining outcomes. Specifically, for actors 1 and 2, the utility possibilities set for their relationship $F(u_1, u_2)$, and their respective *disagreement* or *threat point* payoffs (d_1, d_2) in the interior of this set, the nonsymmetric Nash bargaining solution consists of the arguments (u_1^*, u_2^*) which maximize the expression $N = (u_1 - d_1)^{\alpha} \cdot (u_2 - d_2)^{1-\alpha}$ on $F(u_1, u_2)$, where $\alpha / (1-\alpha), \alpha \in (0,1)$, can be interpreted as the relative bargaining power of the players.

For example, for an affine utility possibilities frontier $u_2 = \beta - u_1$, the bargaining solution is $u_1^* = \alpha(\beta - d_2) + (1-\alpha)d_1, \quad u_2^* = (1-\alpha)(\beta - d_1) + \alpha d_2$, so that each player receives a weighted

[2] Recent articles include Albert and Meckl (2003), Bhattacharyya and Gupta (2015), Chaudhuri (2016), Cogliano *et al.* (2016), Falch (2001), Fuest and Thum (2001), Jayadev (2007), Julius (2009), Kraft *et al.* (2011), Lechthaler and Snower (2012), Merkl and Schmitz (2011), Montagna and Nocco (2015), Sattinger and Hartog (2013), Tavani (2012, 2013) and Ziesemer (2005).

average of his or her disagreement payoff and the maximum utility residual net of the other player's disagreement payoff, where the weights are determined by the bargaining power parameter. This example exhibits the characteristic result that each player's bargaining payoff is increasing in his or her bargaining power and disagreement payoff and decreasing in the corresponding values of these terms for the other player.

However, the axiomatic foundation of the Nash bargaining solution provides no guidance as to how the parameters of the model are to be interpreted in specific applications. In particular, there is no indication as to how players' relative bargaining power is determined, and it unclear whether players' "threat points" represent autarkic payoffs resulting from a failure to reach agreement, or voluntarily chosen alternative market opportunities. Recognizing the interpretational problems engendered by the axiomatic approach to bargaining analysis, Nash (1953) suggested a research agenda in which given axiomatically-based solutions in cooperative game theory are rationalized by being derived as equilibria for corresponding non-cooperative games in which bargaining outcomes are shown to result from players' strategic responses to given transactional conditions. This proposal has come to be known as the *Nash program* for cooperative-game equilibria.

An important development in this program was contributed by Rubinstein (1982), who derived subgame-perfect equilibrium (SPE) for a class of strategic alternating-offer bargaining game with bargaining costs. Subsequent applications of Rubinstein's approach have established strategic foundations for the Nash bargaining solution, as well as demonstrating the apparent inconsistency of that solution with certain interpretations of disagreement payoffs. In particular, Binmore, Rubinstein, and Wolinsky (1986) demonstrate two distinct forms of the alternating-offer bargaining game which yield SPE payoffs corresponding to the symmetric Nash solution.

One form interprets threat-point payoffs as "status quo" or "no loss-no gain" outcomes resulting from the failure to reach agreement in the bargaining relationship. In the other form, the cost of disagreement derives from exogenously given probabilities that the bargaining relationship terminates in any period that agreement is not reached. As they note, in neither case can "disagreement" payoffs be interpreted as payoffs to players' *voluntary* choice to leave the bargaining relationship. They note that the consequence of incorporating outside options is instead to place lower bounds on players' equilibrium payoffs.

In applying the strategic bargaining model to labor market analysis, "outside" or alternative payoffs (whether interpreted as the consequence of exogenous termination of or voluntary exit from given bargaining relationships) can be endogenized by embedding the relevant version of the Rubinstein sequential bargaining game in a matching model that specifies the procedure by which prospective transaction partners are paired. "Competitive" outcomes can then be derived within this matching and bargaining framework as limiting cases as matching frictions approach zero. For example, Rubinstein and Wolinsky (1985) embed a version of the "exogenous termination" model with discounting in a matching framework, enabling them to link equilibrium outside payoffs to relative numbers of participants on both sides of the market and the value of the discount factor assumed to be common to all players.

In earlier work (Skillman 2016, 2018) I study how the voluntary-exit version of the sequential bargaining model can be incorporated in a matching framework to yield equilibrium values of outside options. In the present paper, I embed the "status-quo payoff" bargaining model with outside options in a matching framework in order to study the relevance of the Nash bargaining solution in labor market contexts where parties have outside options, the determinants of relative bargaining power with and without binding outside option constraints, and the determination of bargaining outcomes when outside option payoffs are determined by market competition. The chief findings of the paper's analysis are as follows:

(1) When outside options are absent, equilibrium payoffs correspond to the nonsymmetric Nash bargaining solution in a manner depending on the form of individual bargaining costs. If these costs take the form of time discounting, players' discount rates determine relative bargaining power, while fixed bargaining costs instead affect players' threat-point payoffs.

(2) In the bargaining game with exogenously given outside option payoffs, the Nash bargaining solution again obtains, but with threat points determined by players' outside payoffs. If both players' outside payoffs are low relative to their "status quo" payoffs, the Nash bargaining solution obtains with threat points determined by the latter terms. If both players' outside payoffs are relatively high, the Nash bargaining solution again obtains, but with disagreement payoffs determined by players' outside options. In the asymmetric case in which only one player's outside payoff is relatively high, payoffs are determined by that player's outside payoff and the other player's status quo payoff.

(3) When outside payoffs are endogenously determined in a sequential matching and bargaining framework, agents' bargaining power depends on their probabilities of being rematched after exiting given relationships. If both agents' matching probabilities are low, relative bargaining power is determined solely by internal factors such as agents' relative discount rates. If matching probabilities are both sufficiently high, relative bargaining power is determined by both agents' matching probabilities. In the asymmetric case that only one agent's matching probability is high, only those matching prospects influence agents' relative bargaining power.

(4) A link between steady-state matching and bargaining outcomes and Walrasian equilibrium arises in the case that matching probabilities are asymmetric in the sense that agents on the short

side of the market are sure of being matched, save for the matching friction. In this "quasi-Walrasian" scenario, the Walrasian equilibrium emerges in the limit as matching frictions approach zero.

The paper is structured as follows. In the next section, I present the basic alternating-offer bargaining model with discounting used throughout the paper, excluding outside options. The unique SPE of this bargaining model under two scenarios, differential discount factors and differential fixed costs with a common discounting factor, is characterized and related to the non-symmetric Nash bargaining solution in section 3. In section 4, I introduce outside options and show how this addition modifies the structure of SPE payoffs. Section 5 pursues this point further by embedding the bargaining model with outside options in a variant of the Rubinstein-Wolinsky matching and bargaining model and exploring the consequences of allowing matching frictions to approach zero. In the final section, I discuss relevant directions for future research.

2. The Bargaining Model

In this section, I lay out the basic bargaining model to be used throughout, reserving the incorporation of outside options for later in the paper.

2.1 *Agents, preferences, and payoff possibilities*

Let there be two types of agents, denoted by K and L and indexed by subscript i. [3] Let subscript $-i$ denote "the agent other than i" and let subscript j be used to denote a specific agent. Suppose that agents are infinitely-lived, with time advancing in discrete steps indexed by t. Let τ be used to indicate a given time period. In this section, I'll consider a bargaining relationship between a representative pair of K- and L-type agents.

[3] Where there is no risk of confusion, I shall also use K and L to refer to representative agents of each type.

Suppose that the payoffs of type-*i* agents at any period τ are given by the present discounted value of income flows from that time forward, expressed as $\pi_i = \sum_{t=\tau}^{\infty} \delta_i^{t-\tau} y_{it}, \quad i = K, L$, where y_{it} denotes the net income received by an agent of type *i* in period *t* and $\delta_i \in (0,1)$ represents the time discount factor for an agent of type *i*.

Each agent of type *i* has an endowment of productive assets generating an income of $a_i > 0, i = K, L$, with a corresponding present discounted value of lifetime autarkic income, starting from any period τ, given by $A_i = \sum_{t=\tau}^{\infty} \delta_i^{t-\tau} a_i = a_i / (1-\delta_i)$. To capture the idea that *K*-type agents have greater wealth, assume that $a_K > a_L$.

Now suppose that the production relationship ensuing from a successful pairing of *K*- and *L*-type agents yields a value per period, net of input costs, equal to $v > 0$. The instantaneous *surplus* generated by this production relationship relative to what the agents could otherwise secure for themselves is thus given by $s = v - (a_K + a_L)$. I'll assume that any pairing of *K*- and *L*-type agents generates a strictly positive surplus, so that $v > (a_K + a_L)$.

Suppose for the sake of simplicity that a given production relationship continues forever once begun, and let the corresponding discounted present values of an infinite stream of *v* per period for an agent of type *i* be denoted by $V_i = v / (1-\delta_i)$. The corresponding payoff possibilities frontier for this production relationship is correspondingly given by $\pi_K = V_K - \Delta_{KL} \pi_L$, where $\Delta_{LK} = (1-\delta_K) / (1-\delta_L)$ represents the marginal rate of payoff transformation between *L* and *K*.

2.2 *The bargaining process*

Suppose that a given pair of K and L agents determine the distribution of prospective production income between them by an alternating-offer bargaining process similar to that studied by Rubinstein (1982). Let the agent making the initial offer be determined by the toss of a fair coin[4], after which agents alternate in making offers as long as bargaining continues. In each period, the agent receiving an offer can respond in either of two ways: accept the offer, in which case bargaining immediately concludes forever and the agents share the payoffs according to the accepted proposal, or reject the offer, in which case both agents immediately receive their respective autarkic payoffs and the rejecting agent makes a counter-offer in the following period. Bargaining continues until some agent's offer is accepted; if this never occurs, the agents' payoffs are their respective discounted autarkic income streams (A_K, A_L). Note that since agents have the option of rejecting the other's offers forever, each agent $i = K, L$ can secure a payoff of *at least* A_i and *at most* $V_i - \Delta_{-i,i} A_{-i}$, where $\Delta_{-i,i} = (1-\delta_{-i})/(1-\delta_i)$.

Following Rubinstein, the bargaining costs per period of type-*i* agents are represented by their discount factor $\delta_i, i = K, L$, reflecting the agents' degree of time preference or "impatience." By choosing to reject a standing offer, an agent thus incurs the utility cost of delaying by at least period the distribution of prospective gains from the production relationship. To highlight the distinct implications of representing bargaining costs as discounts on the available surplus, I'll also discuss a version of the model with differential *fixed* bargaining costs $c_i, i = K, L$.

Note that the bargaining game has a stationary structure after the initial coin toss to determine who makes the first offer. Every two periods, the same subgame arises, with the same

[4] I introduce this assumption to distinguish other sources of bargaining power from the well-known "first-mover advantage" enjoyed by initial proposers in sequential bargaining (demonstrated in the proof to Proposition 1).

order of moves, number of remaining periods, autarkic payoffs, and surplus to be distributed. As will become clear, this stationarity plays a central role in the derivation of equilibrium.

A *strategy* for a given agent *i* specifies what action that agent would take at each point in the bargaining game that he or she is called upon to make a move (i.e., to make an offer or respond to one with acceptance or rejection), contingent where relevant on the moves chosen by the other player. Let Z_i represent the set of possible strategies for player *i*, and correspondingly let $Z = Z_K \times Z_L$ denote the set of all possible strategy *combinations* (z_K, z_L).

The bargaining game is therefore completely described by the value to be shared (*v*) plus the players' discount factors (δ_K, δ_L) and possible strategy combinations Z. Let this game be denoted by Γ (and for later reference, let the alternative game characterized by fixed bargaining costs and a common discount factor be denoted by Γ_c). Let a given pair of equilibrium payoffs for the bargaining subgame in which player $j = K \text{ or } L$ makes the first offer be expressed as (π_K^{j*}, π_L^{j*}), and let the corresponding *expected* equilibrium payoffs for the bargaining game be given by (Π_K^*, Π_L^*), where $\Pi_j^* = (\pi_j^{K*} + \pi_j^{L*})/2,\ j = K, L$, the probability-weighted average of the subgame-specific payoffs.

2.3 *Subgame-perfect equilibrium*

I follow Rubinstein (1982) in requiring that bargaining strategies satisfy the condition of *subgame-perfect equilibrium* (SPE), defined as a pair of strategies whose elements constitute a Nash equilibrium for *every subgame* of the overall game Γ. The force of this requirement is that agents' strategic choices must be *credible* in the sense that commitments to *future* moves are optimal to carry out at the time that a given agent must act on such commitments.

3. Strategic foundations of the Nash Bargaining Solution

In this section, I use the strategic bargaining model to link agents' individual bargaining costs to elements of the Nash solution for the present model. Since exit is not a strategic option in this game, it is perhaps best thought of as capturing a situation of effective bilateral monopoly, such as in the case of centralized collective bargaining between capital and labor. I present the subgame-perfect equilibrium for the bargaining game, then discuss the implications of this equilibrium for possible connections between differences in agent wealth and bargaining outcomes.

Proposition 1 The bargaining game Γ has a unique subgame-perfect equilibrium in which bargaining concludes immediately and yields expected payoffs $(\Pi_K^* = x^*, \Pi_L^* = V_L - \Delta_{KL} x^*)$, where $x^* = \alpha \cdot (V_K - \Delta_{LK} A_L) + (1-\alpha) \cdot A_K$ and $\alpha = [(1+\delta_K)(1-\delta_L)] / 2(1-\delta_K \delta_L)$. These payoffs correspond to the unique solution to the power-weighted Nash bargaining problem $\underset{x}{Max}\left(x - A_K\right)^{\alpha} (V_L - \Delta_{KL} x - A_L)^{1-\alpha}$. The equilibrium bargaining payoff of agent $i = K, L$ is increasing in δ_i and a_i and decreasing in δ_{-i} and a_{-i}.

Proof: Consider a representative subgame of Γ immediately following the random selection of the agent making the initial offer in the initial bargaining period $t = 0$. Denote this agent by 1 and the responding agent by 2, with corresponding respective discount factors (δ_1, δ_2) and alternative payoffs (a_1, a_2). Note given these values, the periodic gross surplus *v*, and the alternating-offer structure of bargaining, this subgame is recursive such that the subgame beginning at any even period is identical to the subgame beginning at period 0.

Assume that a SPE exists for this subgame and let *m* denote the *infimum* of the set of equilibrium payoffs for agent 1. Then this is the *least* payoff that agent 1 can expect at period $t = 2$, given the recursive structure of the subgame. In turn, the *least* payoff that 1 can expect from rejecting agent 2's offer in period t =1 is $a_1 + \delta_1 m$. Consequently, the *most* that agent 2 can hope to receive at $t = 1$ is the residual $V_2 - \Delta_{12}(a_1 + \delta_1)$, and thus the most that agent 2 can expect to receive after rejecting agent 1's offer in period t = 0 is $a_2 + \delta_2(V_2 - \Delta_{12}(a_1 + \delta_1))$. It follows that by offering this amount to agent 2, agent 1 can expect to receive at least $V_1 - \Delta_{21} \cdot [a_2 + \delta_2(V_2 - \Delta_{12}(a_1 + \delta_1 m))]$, which is therefore the value of *m*.

Now let *m* denote the *supremum* of the set of equilibrium subgame payoffs for agent 1 and repeat the previous steps of the argument, replacing "least" with "most" and vice-versa. This yields the implication that the infimum and supremum of the set of equilibrium payoffs for agent 1 are equal, so that if a SPE exists for this game, it must yield the unique payoff to player 1 that satisfies the condition $m = V_1 - \Delta_{21} \cdot [a_2 + \delta_2(V_2 - \Delta_{12}(a_1 + \delta_1 m))]$. This expression implies that $m = [(1-\delta_2)(V_1 - \Delta_{21} A_2) + \delta_2(1-\delta_1)A_1] / (1-\delta_1\delta_2)$. Player 2 correspondingly receives the unique payoff $V_2 - \Delta_{12} m = [\delta_2(1-\delta_1)(V_2 - \Delta_{12} A_1) + (1-\delta_2)A_2] / (1-\delta_1\delta_2)$.

It is readily shown that these payoffs can be supported by SPE strategies in which agent 1 always offers agent 2 $V_2 - \Delta_{12} m$ and accepts offers from agent 2 if and only if they at least equal $a_1 + \delta_1 m$, while agent 2 offers $a_1 + \delta_1 m$ in any odd period and only accepts offers from agent 1 at least equal to $V_2 - \Delta_{12} m$. It follows that there is a unique SPE for the overarching game Γ in which each agent receives, with equal probabilities, the respective SPE payoffs of the first and second offerers, implying in turn that the expected payoffs $(\Pi_K^* = x^*, \Pi_L^* = V_L - \Delta_{KL} x^*)$ to K and L agents are as expressed in the proposition.

It is easily shown that these payoffs correspond to the unique solution of the power-weighted Nash bargaining problem, such that $x^* = \arg\max[\left(x - A_K\right)^{\alpha} (V_L - \Delta_{KL} x - A_L)^{1-\alpha}]$, and that each agent's expected bargaining payoff is strictly increasing in that agent's discount factor and alternative payoff, and strictly decreasing in the other agent's discount factor and alternative payoff, completing the proof. ■

Remark: In the case that $\delta_K = \delta_L = \delta$, $\alpha = 1/2$ and the SPE payoffs correspond to those of the symmetric Nash bargaining solution, with $\Pi_K^* = x^* = (V + A_K - A_L)/2$ and $\Pi_L^* = V - x^* = (V + A_L - A_K)/2$.

Proposition 1 provides a strategic foundation for asserting possible theoretical linkages between wealth inequality and economic power, and for the contingent relevance of applying the non-symmetric Nash bargaining solution to this purpose. On the latter point, the proposition shows that the unique SPE payoffs for the strategic bargaining game correspond to the Nash bargaining solution associated with the given payoff possibilities set, threat point (A_K, A_L), and bargaining power parameter α . More precisely, as is made clear in the proof of the proposition, the SPE payoffs are determined by the agents' respective periodic flows of alternative payoffs a_i and per-period discount factors $\delta_i, i = K, L$.

These elements suggest in turn two channels through which wealth differentials might affect equilibrium bargaining outcomes. Most directly, differences in wealth affect bargaining payoffs if these monotonically affect players' alternative payoff flows a_i . In this case, each agent's payoff is increasing in his or her wealth level and decreasing in that of the other player. This would be the case, for example, if agents must rely primarily on their personal economic resources until negotiations are concluded and production begins.

Alternatively, wealth differences would affect bargaining outcomes if it were the case that individual time preferences are wealth-elastic, so that wealthier agents exhibit more patient time preferences. In this case, since each agent's SPE payoffs are increasing in his or her discount factor and decreasing in the other agent's, superior wealth translates into more favorable outcomes of bargaining. The econometric and experimental evidence on this question, while far from conclusive, does suggest that individuals with higher income or wealth have more patient time preferences (see, for example, Lawrance 1991, Samwick 1998, and Harrison *et al*. 2002). In the current two-player context, assuming wealth-elastic time preferences implies that the agent with higher wealth would also enjoy higher bargaining power.

The results discussed thus far have been premised on the assumption that bargaining costs are the consequence of impatient time preferences, which manifest themselves in proportional discounts of prospective surpluses. I next consider the bargaining equilibrium that would instead result if, subject to a minor caveat, agents incurred only *fixed* bargaining costs $c_i > 0, i = K, L$ for each round of negotiations. The caveat is prompted by Rubinstein's (1982) analysis of the scenario in which such fixed costs are the *only* source of bargaining costs for the agents. As he shows, this case yields the stark and rather implausible results that (1) the agent with the smaller fixed bargaining cost receives at least the entire surplus net of the other player's fixed cost, even if the agents' cost differential were vanishingly small, and (2) in the case that agents' fixed bargaining costs are equal, there is a continuum of equilibrium payoffs.

Both of these implications are avoided by the simple and plausible expedient of combining the agents' fixed bargaining costs with a common discount factor δ, which can be set arbitrarily close to one to approximate the case that only fixed bargaining costs are incurred. The implications of this alternative bargaining cost scenario are expressed in the following proposition. The proof, which follows the same steps as that for Proposition 1, is omitted.

Proposition 2 The bargaining game Γ_c has a unique subgame-perfect equilibrium in which bargaining concludes immediately and yields expected payoffs $(\Pi_K^* = x^*, \Pi_L^* = V - x^*)$, where $x^* = (V_K - D_L + D_K)/2$ and $D_i = A_i - [c_i/(1-\delta)]\, i = K, L$. These payoffs correspond to the solution to the symmetric Nash bargaining problem $\underset{x}{Max}\left(x - D_K\right)\cdot(V - x - D_L)$, in which the bargaining payoff of agent $i = K, L$ is increasing in c_{-i} and a_i and decreasing in c_i and a_{-i}.

Note that in this case, bargaining costs do not enter the Nash maximand by determining the exponents on the respective net payoff terms, but rather as shift parameters on the respective threat point payoffs. With that caveat, however, it remains the case that the implications of SPE for the corresponding strategic bargaining game can be represented by a suitably specified version of the Nash bargaining solution.

4. Bargaining with exogenous outside options

In this section, I modify the game previously specified to allow both agents the option of exiting the relationship, then assess the impact of these options on the structure of SPE payoffs. As we'll see, the introduction of outside options alters the determination of "threat point" payoffs.

For the sake of simplicity and focus, I'll assume in what follows that all agents have the same discount factors, but extension to the more general case is straightforward.

4.1 *The bargaining game with outside options*

Let the respective expected outside payoffs to agents *K* and *L* from exiting a given bargaining relationship be given by (W_K, W_L), and assume that all agents have the same parametric discount factor $\delta \in (0,1)$. Since all agents discount future payoffs in the same way, the payoff possibilities frontier simplifies to $\pi_K = V - \pi_L$, where $V = v/(1-\delta)$. In the absence of specific exit barriers, it is reasonable to suppose that $W_i \geq A_i = a_i/(1-\delta)$, since agents could elect to leave the market entirely and receive their autarkic payoff streams. However, the exit payoffs of given agents will in general also depend on their prospects for engaging replacement exchange partners and the payoffs that might be expected from these alternative transactions. Endogenous determination of exit payoffs will be analyzed in the next section; for now, exit payoffs are taken as parametric. I also assume that $V \geq W_K + W_L$, ensuring the viability of individual matches.

4.2 *Equilibrium payoffs and exploitation in the bargaining game with outside options*

The following proposition characterizes equilibrium payoffs for the bargaining game with exit.

Proposition 3. *The bargaining game with outside options has a unique subgame-perfect equilibrium in which initial offers are immediately accepted and yield the respective payoffs* $(\Pi_K^*, \Pi_L^* = V - \Pi_K^*)$, *such that*

(E1) $\Pi_i^* = \left[V - A_{-i} + A_i\right]/2$ *iff* $W_i \leq [\delta(V - A_{-i}) + A_i]/(1+\delta),\ i = K, L$ *;*

(E2) $\Pi_i^* = \left[V - W_{-i} + W_i\right]/2$ *iff* $W_i \geq [\delta(V - A_{-i}) + A_i]/(1+\delta),\ i = K, L$ *; and*

(E3) $\Pi_j^* = \left[(1-\delta)(V - A_{-j}) + (1+\delta)W_j\right]/2$ *iff* for some *j*, $W_j \geq [\delta(V - A_{-j}) + A_j]/(1+\delta)$ *and* $W_{-j} \leq [\delta(V - A_j) + A_{-j}]/(1+\delta)$.

*Proof***:** Let W_j denote the expected payoff to agent j upon exiting the existing bargaining relationship, $j = K \text{ or } L$, and follow the same initial steps as in the proof to Proposition 1 up to the step in which agent *j*'s options at *t* = 1 are considered. At this point, agent *j* can elect to either reject the standing offer and make a subsequent counteroffer, as before, or exit the relationship entirely, and thus can ensure a payoff of *at least* $\max\left\{W_j, a_j + \delta m\right\}$. Player *-j* can in turn expect to receive *at most* $V - \max\left\{W_j, a_j + \delta m\right\} = \min\left\{V - W_j, V - a_j - \delta m\right\}$ if period 1 is reached. It follows immediately that *-j* can thus expect to receive *at most* $\max\left\{W_{-j},\ a_{-j} + \delta \min\left\{V - W_j, V - a_j - \delta m\right\}\right\}$ at *t* = 0, and thus *j* can thus expect to receive *at least* $V - \max\left\{W_{-j},\ a_{-j} + \delta \min\left\{V - W_j, V - a_j - \delta m\right\}\right\}$

$= \min\left\{V - W_{-j},\ (1-\delta)V - a_{-j} + \max\left[\delta W_j, \delta a_j + \delta^2 m\right]\right\}$ at *t* = 0, which therefore equals *m*.

Now let *m* denote the *supremum* of possible (expected) payoffs received by *A* in any equilibrium and repeat the foregoing argument, replacing *at least* with *at most* and vice-versa. As in the proof for Proposition 1, this yields the same value for *m*, implying that if an equilibrium exists for this subgame, it yields a unique pair of payoffs. It is then straightforward to show that these payoffs can be supported by a pair of subgame-perfect strategies in which player *j* proposes to receive *m* whenever making an offer, and accepts any counter offer yielding at least $z = \max\left\{W_j,\ \delta_j m\right\}$; correspondingly, player *-j* proposes *z* for *j* whenever making a counteroffer, and accepts any offer of at least $V-m$. These strategies are subgame-perfect and ensure that bargaining concludes immediately in equilibrium.

There are thus three possible equilibrium scenarios for the subgame in which player *j* makes the initial offer, depending on the pattern of inequalities among the three terms in the expression for m. For example, $m=(1-\delta)V-a_{-j}+\delta a_j+\delta^2 m$ if $V-W_{-j}\geq(1-\delta)V-a_{-j}+\delta a_j+\delta^2 m$, $\geq(1-\delta)V-a_{-j}+W_j$, implying in turn that $m=[(V-A_{-j})+\delta A_j]/(1+\delta)=\pi_j^{j*}$. This is the share the initial offerer proposes for him- or her*self* given that *B* cannot credibly threaten to exit in response to an unfavorable offer. Correspondingly, $V-m$, the share received by player *-j*, equals $[\delta(V-A_j)+A_{-j}]/(1+\delta)=\pi_{-j}^{j*}$. Back substitution of the value of *m* into the inequality conditions establishes that this equilibrium case occurs if and only if $[(V-A_{-i})+\delta A_i]/(1+\delta)\geq W_i\ \forall i$, which is the condition for scenario (E1) of the proposition, which occurs when neither player's outside payoff is sufficiently high for the threat of exit to induce a higher payoff. Since this outcome occurs no matter which agent makes the initial offer, and agents have equal probabilities of making the initial offer, the expected payoff of each player in equilibrium scenario (E1) is $\Pi_i^*=[\pi_i^{i*}+\pi_i^{-i*}]/2=[V-A_{-i}+A_i]/2,\ i=K,L$, as asserted.

The conditions for the remaining two scenarios are established in similar fashion by considering the implications of the other possible combinations of inequalities. The key difference of these latter cases from the one examined above is that at least one player has a sufficiently high outside payoff to make exit a credible threat for that player in response to an unfavorable offer from the other. ■

The sense of Proposition 3 is that the introduction of outside options gives rise to three distinct equilibrium payoff scenarios, depending on the magnitudes of the agents' outside payoffs relative to what they could achieve in the bargaining game without exit. These three scenarios can be summarized as follows:

(E1): If both agents' outside payoffs are no greater than what they could respectively secure by rejecting the opponent's offer in the bargaining game without exit, then each agent's equilibrium payoff is as described in Proposition 1 (given equal discount factors).

(E2) If both agents' outside payoffs are greater than what they could respectively secure by rejecting the opponent's offer in the bargaining game without exit, then each agent's equilibrium payoff is strictly increasing in their own outside payoff and strictly decreasing in the opposing agent's outside payoff. The scope for this equilibrium scenario is limited, in the sense that it requires the discount factors of both agents to be sufficiently below one.

(E3) If one agent's outside payoff is relatively high in the sense described in (E2) and the other agent's outside payoff is relatively low in the sense described in (E1), then the payoff of the first agent is strictly increasing in their outside payoff and strictly decreasing in the status quo payoff of the latter agent, while the expected payoff of the other agent is strictly increasing in their status quo payoff and strictly decreasing in the exit payoff of the opposing agent.

Note with respect to all of these scenarios that no necessary connection is presumed between status quo and outside payoffs, since the latter may be influenced by individual agents' alternative transaction opportunities within the market. Consequently, the possibility of exit places a "floor" on agents' equilibrium bargaining payoffs. Experimental and econometric evidence for the effects of asymmetric outside options on bargaining outcomes is respectively presented by Binmore *et al*. (1989) and Scaramozzino (1991).

5. Bargaining with endogenous outside options in a market with sequential matching

5.1 *The matching process*

Now suppose that the bargaining game with outside options studied in the previous section is embedded in a market process in which the *K*- and *L*-type agents are randomly matched with probabilities determined by their respective numbers in the market. This step makes it possible to link agents' outside payoffs endogenously to underlying market conditions as well as to autarkic income flows based on agents' respective wealth holdings. This matching and bargaining framework is studied by Rubinstein and Wolinsky (1985) using a bargaining model in which given transactions are assumed to be terminated exogenously with some positive probability, without the voluntary exit options assumed in the previous section. A corresponding matching and bargaining framework with voluntary exit is analyzed in Skillman (2016). Here, I extend that framework to incorporate "status quo" payoffs in individual bargaining relations.

As in Rubinstein and Wolinsky (1985), the market mechanism pairing agents of different types is represented as a probabilistic matching process based on net agent flows per period. Let $N_{it} > 0$ represent the number of type-*i* agents in the market at time *t*. Then the number of matches of *L* and *K* agents in any period *t* is given by a *matching function* $M_t = m(N_{Kt}, N_{Lt})$, where M_t denotes the number of matches of *K* and *L* agents in period *t* and the function *m* is assumed to be non-decreasing in its arguments such that $M_t \le Min\{N_{Kt}, N_{Lt}\}$. The corresponding probability that an agent of type *i* will be matched with an agent of complementary type in period *t* is indicated by $q_{it} = m(N_{Lt}, N_{Kt}) / N_{it}$.

In any period τ, then, an unmatched agent has the probability $q_{i\tau}$ of being matched and engaging in bargaining with an agent of complementary type. With corresponding probability

$1-q_{i\tau}$, the agent remains unmatched, receiving an immediate autarkic payoff of a_i and then re-entering the matching process in the following period. Letting $W_{i\tau}$ denote the expected present payoff of an agent who is unmatched at the beginning of period τ, it follows that an unmatched agent has an expected payoff of $a_i + \delta_i W_{i,\tau+1}$.

Consequently, the value of an agent's exit option in a given period τ is expressed by $W_{i\tau} = q_{i\tau}\Pi^*_{i\tau} + (1-q_{i\tau})\left[a_i + \delta W_{i,\tau+1}\right]$, where $\Pi^*_{i\tau}$ denotes an equilibrium bargaining payoff for agent *i* in period τ and $W_{i,\tau+1}$ denotes the expected payoff to an agent of the same type who is unmatched at the beginning of the following period. Recall from Proposition 3, however, that bargaining payoffs depend in turn on the values of outside options in two equilibrium scenarios. Thus, to close the system and yield determinate outcomes, a condition is needed to link current to future exit payoffs. For the overall matching and bargaining game, I follow Rubinstein and Wolinsky in limiting attention to *semi-stationary* strategies, which requires that (1) all agents of the same type play the same strategies, (2) agents of each type play the same strategy in any match, and (3) there is a *steady state* in agent flows such that for each *i*, $N_{it} = N_i$.

Now consider how agents' outside payoffs are determined by the market matching process in a semi-stationary equilibrium. Denoting the expected steady-state payoff of a newly-matched agent by Π^s_i and the expected steady-state payoff of a currently unmatched agent by W^s_i, $i = K, L$, it follows that the value of the latter in a semi-stationary equilibrium is given by $W^s_i = q_i\Pi^s_i + (1-q_i)\left[a_i + \delta_i W^s_i\right]$, implying in turn that the steady-state value of an unmatched player's expected payoff is expressed as

(1) $$W^s_i = [q_i \cdot \Pi^s_i + (1-q_i)(1-\delta)\cdot A_i]/(1-\delta_i(1-q_i)),\ i = K, L,$$

where $A_i = a_i / (1-\delta_i)$.

Expression (1) indicates that the expected payoff to an unmatched player depends on the probability-weighted average of the bargaining payoffs from being successfully matched and from autarkic income in a single period, where the probability weights are determined by the matching function and steady-state player flows.

The steady-state equilibrium payoff to a matched player $i, i = K, L$, is then determined by the condition that

(2) $$\Pi_i^s = \Pi_i^* \Big| W_j = W_j^s,\ j = K, L ,$$

implying that outside and bargaining payoffs are simultaneously determined in equilibrium.

5.2 *Steady-state bargaining outcomes*

In general, all three of the bargaining scenarios described in Proposition 3 can be sustained in the semi-stationary market equilibrium for suitable values of matching probabilities and agents' discount factors, with the added condition that outside payoffs are determined endogenously in the respective bargaining equilibrium scenarios by autarkic income flows and matching probabilities. Equilibrium bargaining payoffs with endogenously determined outside option values are characterized in the following proposition. To facilitate the expression and proof of this result, let *M* denote the player type with the greater matching probability, given by $q_M = Max\{q_K, q_L\}$, and let the matching probability of the other player type be given by q_m. In the case that matching probabilities are equal, either player type can be designated as *M*.

Proposition 4 *In the matching and bargaining game with common discount factor* $\delta \in (0,1)$, *steady-state equilibrium payoffs take the form of the Nash bargaining solution*

$\Pi_i^* = \alpha_i \cdot (V - A_{-i}) + (1-\alpha_i) \cdot A_i$, $\Pi_{-i}^s = V - \Pi_i^s$, $i = K, L$, *such that*

(E1) *bargaining parameter* α_i *is determined independently of players' matching probabilities if*

$q_M / 2[1-\delta(1-q_M)] \leq \delta / (1+\delta)$;

(E2) *bargaining parameter* α_i *is increasing in player i's matching probability and decreasing in player –i's matching probability iff*

$q_m(1-q_M) / [(1-q_m) + (1-q_M) - 2\delta(1-q_m)(1-q_M)] \geq \delta / (1+\delta)$; and

(E3) *bargaining parameter* α_M *is increasing in M's matching probability and is independent of m's matching probability if* $q_M / (2-q_M) \geq \delta / (1+\delta)$ *and*

$q_m(1-q_M)(1+\delta) / [(2-q_M)(1-\delta(1-q_m))] \leq \delta / (1+\delta)$

Proof: For each equilibrium scenario, bargaining payoffs are as given in Proposition (3) subject to steady-state conditions (1) and (2). This yields a system of four equations in four steady-state variables, $(\Pi_K^s, \Pi_L^s, W_K^s, W_L^s)$, along with the inequality conditions defining each scenario. In every case, the steady-state value of any player *j*'s outside payoff takes the form

$W_j^s = \beta_j \cdot (V - A_{-j}) + (1-\beta_j)A_j$, $j = K, L$, where $\beta_j \in (0,1)$ is specific to the equilibrium scenario. Thus, the inequality relation defining each scenario reduce to a comparison of β_j and $\delta / (1+\delta)$.

Now consider the respective equilibrium scenarios. For (E1), $\Pi_i^s = \left[V - A_{-i} + A_i\right] / 2$ $\forall i$, where the power coefficient $\alpha = 1/2$ is independent of players' matching probabilities. This scenario obtains just in the case that

$W_i^s = [q_i \cdot (V - A_{-i}) + (2(1-q_i)(1-\delta)]A_i)] / [2(1-\delta(1-q_i))] \le [\delta(V - A_{-i}) + A_i] / (1+\delta)\ \forall i,$ which is equivalent to the condition $\beta_M = q_M / [2(1-\delta(1-q_M))] \le \delta / (1+\delta)$ as stated.

For (E2), $\Pi_i^s = \left[V - W_{-i}^s + W_i^s\right]/2 = \alpha_i (V - A_{-i}) + (1-\alpha_i) A_i$, where the power coefficient $\alpha_i = (1-q_{-i})(1-\delta+\delta q_i) / [(1-q_i) + (1-q_{-i}) - 2\delta(1-q_i)(1-q_{-i})]$ is increasing in player i's matching probability and decreasing in player –i's matching probability. This scenario obtains just in the case that $\beta_i = q_i \cdot (1-q_{-i}) / [(1-q_i) + (1-q_{-i}) - 2\delta(1-q_i)(1-q_{-i})] > \delta / (1+\delta)\ \forall i$, which is equivalent to the condition that $\beta_m > \delta / (1+\delta)$, as asserted.

For (E3), $\Pi_M^s = \left[(1-\delta)(V - A_{-j}) + (1+\delta) W_M^s\right]/2 = \alpha_M \cdot (V - A_m) + (1-\alpha_M) A_M$, where the bargaining power coefficient of the player with the highest matching probability, $\alpha_M = (1-\delta(1-q_M)) / (2-q_M)$, is increasing in q_M and independent of $q_m < q_M$. This scenario obtains just when $W_M^s = [q_M (V - A_m) + 2(1-q_M) A_M] / (2-q_M) \ge [\delta(V - A_m) + A_M] / (1+\delta)$ and $W_m^s = [q_m (1-q_M)(1+\delta)(V - A_M) + (1 + (1-q_M)(1-q_m) - \delta(2-q_M-q_m) A_m] / (2-q_M)(1-\delta+\delta q_m)$ $\le [\delta(V - A_M) + A_m] / (1+\delta)$, which is equivalent to the conditions $\beta_M = q_M / (2-q_M) > \delta / (1+\delta)$ and $\beta_m = q_m (1-q_M)(1+\delta) / (2-q_M)(1-\delta(1-q_m)) \le \delta / (1+\delta)$. ■

Proposition 4 establishes, in the context of this matching and strategic bargaining framework, the general validity of the Nash bargaining solution when outside options are endogenously determined in the steady-state equilibrium. The key insight is that since outside payoffs themselves depend on combinations of bargaining and autarkic payoffs, steady-state bargaining payoffs must in turn depend on weighted combinations of transactional value added and autarkic payoffs. These bargaining weights, however, depend on the bargainers' matching probabilities.

There are three equilibrium scenarios depending on the relation of these probabilities to a benchmark set by players' ability to credibly reject a range of offers without resorting to exit. Equilibrium scenario (E1) obtains when both players' prospects of being rematched in a given period are relatively low. In this case, bargaining weights in the Nash bargaining solution are determined independently of players' matching probabilities, and thus will in the general case depend on the values of the agents' discount factors (here assumed to be identical).

In equilibrium scenario (E2), in contrast, arises when both players' matching probabilities are high relative to the benchmark. For example, if the players' matching probabilities are identical, then under the specification of the model assumed in the proposition, this scenario arises when the common value of the matching probabilities in a given period is two-thirds. In this case, each player's bargaining power parameter is increasing in the player's own matching probability and decreasing in the matching probability of the other agent. Finally, scenario (E3) arises in the asymmetric case that only one player's matching probability is relatively high. In that case, the bargaining power parameter of each player is a function of only the matching probability of the favored player.

Thus, the proposition provides a foundation for testable hypotheses regarding the determination of relative bargaining power under alternative market conditions. One might expect scenario (E3) to arise, for example, only at extreme stages of the business cycle, characterized by either a relative excess of job vacancies or labor unemployment rates.

5.3 *Equilibrium with quasi-Walrasian matching prospects*

A featured implication of Rubinstein and Wolinsky's matching and bargaining analysis is that steady-state payoffs do not converge to "competitive equilibrium" levels in the limit as market frictions (represented by agent discount rates or length of bargaining and matching periods) approach zero. Specifically, their model does not yield the characteristic Walrasian result that all gains from trade flow to agents on the "short side" of the market (Rubinstein and Wolinsky 1985, pp. 1147-1148). As can be seen from Proposition 4, steady-state outcomes in the present model, which is adapted from their framework, yield the same implication: if matching probabilities are bounded below 1, both agents share in the surplus as the common discount factor approaches one.

However, aside from payoff loses associated with time discounting, one might also consider the inability of agents on the short side of the market to be matched with certainty a manifestation of exchange frictions. Assume, therefore, that the matching function takes the specific form $M_t = \min\{N_{it}, N_{jt}\}(1-\varepsilon)$, with $\varepsilon > 0$. I refer to this matching function as *quasi-Walrasian* because it captures the asymmetry that, save for the presence of matching frictions, agents on the short side of the market are certain of being matched.[5] The implications of this assumption are expressed in the final proposition.

Proposition 5. *Suppose that the agents' common discount factor is bounded below one,* $\forall t\ N_{it} = N_i^s > 0, i = K, L$, *and the matching function is given by* $M_t = \min\{N_{it}, N_{jt}\}(1-\varepsilon)$. *Then in the limit as* $\varepsilon \to 0$, *semi-stationary equilibrium yields payoffs* $(\Pi_K^*, \Pi_L^* = V - \Pi_K^*)$ *such that* $\Pi_j^* \begin{cases} = V - A_{-j} \text{ if } N_j^s < N_{-j}^s \\ \in [A_j, V - A_{-j}] \text{ if } N_j^s = N_{-j}^s \end{cases}$, $j = K \text{ or } L$.

[5] The issue in question here goes beyond the specification of the matching function, however. Gale (1987) argues that Rubinstein and Wolinsky's model fails to articulate exchange conditions sufficient to ensure steady-state numbers of agents as successfully matches occur. Building such conditions into Rubinstein and Wolinsky's framework, Gale argues, yields Walrasian equilibrium outcomes in the limit as entry costs approach zero.

***Proof*:** In the case of semi-stationary equilibrium such that $N_j^s < N_{-j}^s$ for a particular agent *j* = *K* or *L*, $q_j = (1-\varepsilon)$ given the quasi-Walrasian matching function, implying in turn that $q_j \to 1$ in the limit as ε approaches zero. In contrast, the same property of the matching function implies that $q_{-j} \leq N_j^s / N_{-j}^s < 1$ for all positive values of $\varepsilon \in [0,1)$ and is thus bounded strictly below 1. Correspondingly, the steady-state value of agent *j*'s expected payoff from exit becomes $W_j^s = [(1-\varepsilon)\Pi_j^s + \varepsilon(1-\delta)A_j] / (1-\delta\varepsilon)$, which approaches Π_j^s in the limit as $\varepsilon \to 0$.

To see that this limit case is inconsistent with equilibrium scenarios (E1) and (E2), assume otherwise. In scenario (E1), This cannot occur in equilibrium scenario (E1), since in this case $\Pi_j^s = [V - A_{-j} + A_j] / 2 \to W_j^s$ as $\varepsilon \to 0$, contradicting the requirement that $W_j^s \leq [\delta(V - A_{-j}) + A_j] / (1+\delta)$ for scenario (E1) to occur. In equilibrium scenario (E2), $\Pi_j^s = [V - W_{-j}^s + W_j^s] / 2 \to V - W_{-j}^s$ as $\varepsilon \to 0$, implying in turn that $\Pi_{-j}^s \to W_{-j}^s$ in the same limit. But then $W_{-j}^s \to [q_{-j}^s W_{-j}^s + (1-q_{-j}^s)(1-\delta)A_{-j}] / (1-\delta(1-q_{-j}^s) = A_{-j}$ as ε approaches zero, which contradicts the condition $W_{-j}^s \geq [\delta(V - A_{-j}) + A_j] / (1+\delta)$ required for scenario (E2) to occur. This leaves equilibrium scenario (E3) , which implies that $\Pi_j^s = \left[(1-\delta)(V - A_{-j}) + (1+\delta)W_j^s\right] / 2 \to V - A_{-j}$ as $\varepsilon \to 0$, implying in turn that Π_{-j}^* approaches A_{-j} as ε approaches zero. It is readily verified that both inequality requirements for scenario (E3) are satisfied for values of ε sufficiently close to zero.

For the case that $N_j^s = N_{-j}^s$, the semi-Walrasian property of the matching function implies that $q_i^s = (1-\varepsilon), i = K, L$, implying in turn that for all agents, $W_i^s \to \Pi_i^s$ as $\varepsilon \to 0$. As before, this limit case is inconsistent with equilibrium scenario (E1), but is compatible with scenarios (E2) or (E3) for values of $\Pi_j^{*=s}$ in the indicated range, given $\Pi_j^s + \Pi_{-j}^s = V$. ■

Proposition 5 establishes a connection between steady-state matching and bargaining outcomes and competitive equilibrium. In the limit as matching frictions approach zero, given the "quasi-Walrasian" matching function adopted here, agents on the long side of the market receive just their status quo income flows which also correspond to their exit payoffs in the steady state, while agents on the short side of the market receive the entire residual. If there are equal numbers of K- and L-type agents in the market, equilibrium payoffs are indeterminate, such that the equilibrium payoff to an agent of a given type ranges from that agent's status quo payoff to the entire production value V net of the other agent's status quo payoff.

6. Concluding comments

Contrary to the impression created by strategic bargaining models à la Rubinstein in which outside option payoffs are exogenously given, the presence of exit options does not necessarily invalidate the application of the Nash bargaining solution to labor market transactions. However, the results presented here should be considered suggestive insofar as the flow from specific features of the matching and bargaining framework presented here. Thus an obvious aim of future research is to establish a broader foundation for these conclusions.

While possible implications of systematic wealth differences between firm owners and workers are discussed, there are other evident asymmetries in the bargaining positions of labor market participants that should be considered in future efforts to tailor the Nash bargaining framework more closely to common features of such markets. Of particular salience is the fact that many if not most bargaining relationships in capitalist firms take the form of "monopoly-oligopoly" transactions, given that firm owners, even if multiple, are collectively represented by a management bargaining team, while workers generally only enjoy this benefit in the context of legally specified collective bargaining arrangements. In this connection, an interesting but open question is whether bilateral monopoly-oligopoly relationships can be captured by an additional refinement of the Nash bargaining solution.

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