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Financial Stability with Sovereign Debt*

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Abstract

Are government guarantees or financial regulation a more effective way to prevent banking crises? I study this question in the presence of a negative feedback loop between the fiscal position of the government and the health of the banking sector. I construct a model of financial intermediation in which the government issues, and may default on, debt. Banks hold some of this debt, which ties their health to that of the government. The government's tax revenue, in turn, depends on the quantity of investment that banks are able to finance. I compare the effectiveness of government guarantees, liquidity regulation, and a combination of these policies in preventing self-fulfilling bank runs. In some cases, a combination of the two policies is needed to prevent a run. In other cases, liquidity regulation alone is effective and adding guarantees would make the financial system fragile.

Keywords: Bank runs, Sovereign default, Feedback loop, Government guarantees, Liquidity regulation

JEL classification: G21, G28, H63

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1 Introduction

The financial crisis of 2007-08 triggered an ongoing debate over how policy makers can most effectively promote financial stability. In particular, there has been much discussion and controversy over whether policy makers should expand government guarantees of the banking system, cut back on these guarantees and focus on financial regulation, or combine guarantees with new regulations. As emphasized by Gorton (2010), a government guarantee in the form of deposit insurance ended the type of banking panics that the U.S. had suffered prior to 1933. However, the Dodd-Frank Wall Street Reform and Consumer Protection Act moves in the opposite direction, introducing regulatory reforms "to protect the American taxpayer by ending bailouts" and restricting the ability of the public sector in the U.S. to provide guarantees in a future crisis. In this paper, I ask whether government guarantees, financial regulation, or a combination of these two policies is most effective at preventing financial crises.

Government guarantees are costly. A sizable literature focuses on the moral hazard problem associated with guarantees, in which the anticipation of government support in bad times distorts the incentives of financial intermediaries.[1] The Irish financial crisis in 2008 illustrates another important cost: guarantees may undermine the solvency of government. When the Irish government announced it would guarantee banks' deposits on September 30, 2008, it took on a liability that was potentially three times the size of annual GDP. The anticipated difficulty of financing this guarantee undermined its credibility, and the cost of credit default swaps increased for both banks and the government.[2] This example shows the fiscal cost of guarantees hurts the sovereign debt sustainability, while an unsustainable debt, in turn, undermines the effectiveness of guarantees. In the years since the crisis, policy makers have been searching for ways to prevent or mitigate this negative feedback loop.[3]

In this paper, I study not only guarantees (as in Konig, Anand, and Heinemann (2014) and

[1]See, for example, Kareken and Wallace (1978), Gale and Vives (2002), Rancière and Tornell (2016), and Keister (2016).

[2]Further details can be found in IMF (2015).

[3]The former IMF managing director Christine Lagarde mentioned "We must break the vicious cycle of banks hurting sovereigns and sovereigns hurting banks" in her speech of January 2012, and Governor Ignazio Visco of the Bank of Italy said "An intensely debated topic in the context of possible further financial reforms... concerns possible actions to address the negative loop between sovereign and bank risk." in his speech of May 2016.

Leonello (2018)), but also financial regulation and the combination of these policies when there is a negative feedback loop between the government and banking sector. In my model, a banking crisis can be transmitted to the government sector through tax revenue. Previous work that studies self-fulfilling bank runs either ignores the tax revenue channel (Konig et al. (2014) and Leonello (2018)) or assumes it does not depend on bank lending (Cooper and Nikolov (2018)). In addition to introducing this channel, my model is the first in this literature to study the effectiveness of liquidity regulation and its interaction with government guarantees.

Are guarantees, financial regulation or a combination of these policies the best way to promote financial stability given this loop? I address this question in a model of financial intermediation based on a version of the classic model of Diamond and Dybvig (1983) extended to include a government that issues, and may default on, debt. Intermediaries invest in a combination of government bonds and long-term projects, and these projects are a source of tax revenue for the government when they mature. As in Diamond and Dybvig (1983) and many others, intermediaries perform maturity transformation and are potentially susceptible to a self-fulfilling run by depositors. The government and banking sector are linked in two ways:

1. Tax revenue: taxes on matured investment financed by intermediaries are an important source of revenue for the government.

2. Bond price: intermediaries hold government bonds and changes in the price of these bonds affect their solvency.

My analysis begins by studying equilibrium outcomes when there are no guarantees or regulation, which I call the no-policy regime. If depositors run on the banking system, banks liquidate projects to redeem withdrawals. If enough depositors withdraw, the banks will be forced to liquidate all of their projects, which implies the government has no tax base and is unable to repay its bonds. I show that, in the no-policy regime, the financial system is always vulnerable to a run and a run always causes the government to default on its debt.

I then introduce three different policy regimes into this model and ask under what conditions each regime eliminates the crisis equilibrium. The first regime is a government guarantee of deposits in the banking system. The role of this guarantee is to prevent liquidation of long-term investment and, in the process, to preserve the tax base. However, its effectiveness depends on whether the government will be able to pay off the debt used to finance the guarantee. I find the guarantee tends to be effective when the return on long-term investment is high and when the government's initial debt is small. The guarantee becomes a third linkage between the government and banking sector:

3. Guarantees: the government may make transfers banks facing a liquidity shortage, increasing the government expenditures.

The second policy regime is a type of liquidity regulation that requires banks to have a minimum level of liquid assets relative to expected short-term outflows. When this requirement binds, a bank must shift its portfolio away from profitable projects and toward bonds. This regulation may prevent a run, but may also distort the allocation and can even cause default if it is too tight. I find the regulation tends to be more effective than the guarantee at preventing financial crises when the return on long-term investment is low, while the guarantee is likely to be more effective when this return is high. In some cases, both policies are effective at preventing a run. In these cases, the guarantee implements a better allocation because it does not distort banks' choices.

Finally, as the third policy regime, I examine the combination of these two policies. Liquidity regulation may complement the guarantee, because it requires banks to have a larger amount of bonds and to reduce their short-term outflows. The banks can redeem some extra withdrawals using the returns from their bond holdings without liquidating long-term projects. Furthermore, liquidity regulation lowers banks' short-term liabilities, which decreases the obligations faced by the government in the event of a run. The combination of these two policies is necessary to prevent a run in some cases. In other cases, liquidity regulation alone is effective and adding guarantees would make the financial system fragile.

The remainder of the paper is organized as follows: Section 2 introduces the environment and defines financial stability and fragility. In Section 3, I present equilibrium outcomes without policy, and I study equilibria given different policy regimes in Section 4. I offer some concluding remarks in Section 5.

2 The Model

The analysis is based on a version of Diamond and Dybvig (1983) model augmented to include a government that issues and may default on debt. I introduce a financial market in which the bonds are traded. This section describes the basic elements of the model.

2.1 The environment

There are three periods, labeled $t = 0, 1, 2$, and a continuum of depositors indexed by $i \in [0, 1]$. Each depositor has preferences given by

$$U(c_1, c_2; \omega_i, \delta, \mathbb{1}_G) = u(c_1 + \omega_i c_2) - \mathbb{1}_D \delta,$$

where c_t is the consumption of goods in period t. The function u is assumed to be logarithmic. The parameter ω^i is a binominal random variable with support $\Omega \equiv \{0, 1\}$, which is realized in period 1 and privately observed by the depositors. If $\omega_i = 1$, depositor i is patient, while she is impatient if $\omega_i = 0$. Each depositor is chosen to be impatient with a known probability $\pi \in (0, 1)$, and the fraction of impatient depositors is also equal to π. The other component of the preferences is the welfare loss associated with a sovereign default. The indicator function $\mathbb{1}_D$ takes the value 1 if the government defaults and 0 otherwise, and δ captures the level of loss. I assume δ is sufficiently large such that a default should be avoided at any cost if possible.[4]

Technologies: Depositors are each endowed with one unit of all-purpose good which can be used for consumption or investment at the beginning of period 0. There is a single, constant-returns-to-scale technology for transforming this endowment into consumption in the last period. A unit of the goods invested in period 0, called a *project*, yields with $R > 1$ in period 2 or $r < 1$

[4] A government default would negatively affect the credibility, institutions, public safety or infrastructure in an economy. This default cost captures all such negative effects. See, for example, Bolton and Jeanne (2007).

in period 1, where r represents the liquidation value.

Government: The government must finance a given level of expenditure in period 0. This expenditure can be interpreted as *initial debt*, and is denoted by $d_0 > 0$. The government issues bonds to raise funds in period 0, and levies taxes on matured projects in period 2 to repay the bonds. The tax rate will be determined by dividing the outstanding debts by the return from the matured projects. The government collects taxes just enough to repay the outstanding debts in period 2. The matured projects can be interpreted as the output of the economy. If the government is unable to collect enough taxes to repay the debts fully, the government defaults on the debts (called as sovereign default), and depositors incur the welfare loss δ.

Bond market and Investors: The government bonds can be traded in a competitive asset market in periods 0 and 1, and depositors and a large number of wealthy risk-neutral investors may purchase them. These investors have endowments in period 1 and preferences are given by

$$v_j(c_2^f) \equiv c_2^f,$$

where c_2^f is the period-2 consumption of investor f. The investors have an outside option that pays a return $R^* \geq 1$ in period 2, and they arbitrage between this outside option and the bonds. Let q_t denote the bond price in $t = 0, 1$. This setup drives that, given that the government is expected to repay the debts fully, the bond is valued as $q_t = \frac{1}{R*}$ by these investors. If the investors anticipate that the government will not repay the debts, the bond is valued as $q_t = 0$.

Financial intermediation: Depositors pool their resources to form a bank in order to insure against liquidity risk, as in Diamond and Dybvig (1983) and many others. This representative bank behaves competitively in the sense that it takes the returns on assets as given and invests in a combination of projects and government bonds. Each depositor can either contact her bank to withdraw funds in period 1 or wait until the final period to withdraw. Depositors are isolated from each others in period 1 and 2, and they cannot trade with each other. Upon withdrawing, the depositor must consume what is given immediately. Repayments follow a *sequential service constraint* (Wallace (1988)), and the order of a depositor's arrival at the central location is randomly determined after they decide to withdraw. Therefore, each depositor learns her type at the beginning of period 1 and decides whether to contact the central location or not. Once a depositor decides to contact the bank, she learns her position in the queue of depositors to withdraw.

2.2 Financial crisis

Withdrawal plan: Depositors may condition their withdrawal decision on an extrinsic *sunspot* signal $s \in S = \{\alpha, \beta\}$. This sunspot variable is realized at the beginning of period 1 and is observable to depositors and investors, but unobservable to banks. A depositor's withdrawal decision depends on both her type and the sunspot signal. Let y_i denote the withdrawal strategy for depositor i such that

$$y_i : \Omega \times S \mapsto \{0, 1\},$$

where $y_i(\omega_i, s) = 0$ corresponds to withdrawal in period 1 and $y_i(\omega_i, s) = 1$ corresponds to withdrawal in period 2. A bank run is defined as withdrawals by a positive measure of patient depositors, and therefore a bank run occurs if $y_i(1, s) = 0$ for a positive measure of patient depositors. Let y denote the profile of withdrawal plans for all depositors.

I assume that the probability agents assign to the bad sunspot state in period 0 is zero. This assumption is a useful and common simplification in the literatures on banking and other financial crises.[5] The occurrence and timing of a financial crisis is notoriously difficult to predict, and there is evidence that the risks of such events are effectively overlooked by private agents in good times. By a continuity argument, the results below can also be shown to hold if this probability is positive but sufficiently small.

2.3 Timeline

The timing is summarized in Figure 1. In period 0, depositors deposit their endowments with the bank in each central location. The government issues the bonds to investors and banks through the bond market, in order to repay the initial debt. The bank divides the resources deposited by

[5] See, for example, Diamond and Dybvig (1983), Chang and Velasco (2000), and Ennis and Keister (2009).

the depositors between bonds and projects, and the period ends. At the beginning of period 1, depositors learn their type ω^i and the sunspot state, and choose whether to withdraw in period 1 or wait. Withdrawing depositors then begin to arrive one at a time at their banks and are served as they arrive. To finance these withdrawals, a bank sells bonds to investors and may liquidate projects. In period 2, the government levies taxes on matured projects and repays its bonds. Banks then repay the remaining depositors.

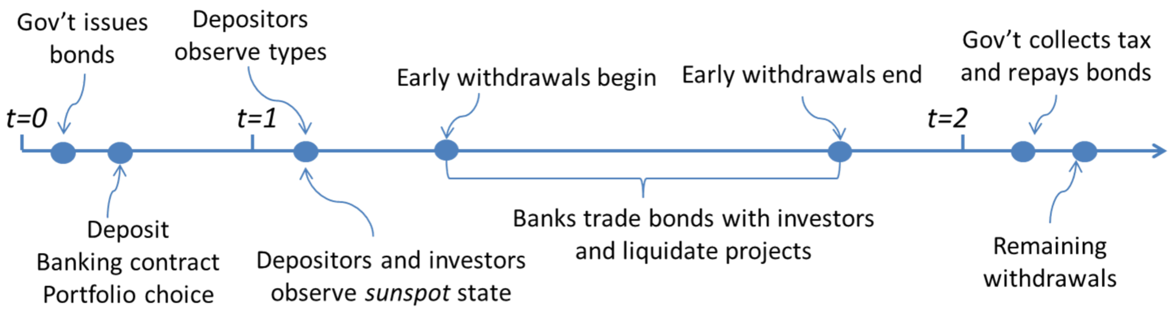

Figure 1: Timeline

2.4 Discussion

No strategic default: I assume that its government fulfills the obligations whenever it can, and it defaults on the debts if and only if repayment is not feasible. A strategic default is a common idea in the literature studying sovereign defaults in emerging economies. It could be, however, too costly for advanced economies to strategically default on its debts. For instance, domestic financial and non-financial firms in advanced economies might consist of a significant part of the global supply chains and global financial markets. Sovereign defaults in advanced economies can, then, be hugely disruptive events for global supply chains and global financial markets. This assumption is commonly used in studying public debt sustainability in advanced economies.[6]

Tax exempted bond: The assumption that the government does not levy taxes on government bonds is an essential ingredient for financial fragility in this environment. If I were to instead allow the government to freely tax bold holdings, it would be able to directly reduce

[6]See, for example, Ghosh, Kim, Mendoza, Ostry, and Qureshi (2013) and Cottarelli, Mauro, Forni, and Gottschalk (2010).

its liabilities to bond holders. In other words, taxing bond holdings would effectively allow the government to indirectly default on its obligations, either partially or in full. In the extreme case of a 100% tax rate on gross returns from the bonds, the government's repayment and the debt holders' tax payment offset each other, and the government has zero net payment to debt holders. My assumption that the government cannot tax bond holdings is equivalent to assuming that the default cost δ is present regardless of whether the default reflects a failure to repay or a direct confiscation of the bonds through taxation.

Alternatively, the government may levy taxes on net returns from bonds. However, the net return must at least equal the risk-free rate R^* because of the investors' financial arbitrage. As long as the risk-free rate is not very high, and the taxes on net return from these bonds would not help the government revenue much. Costs to implement such taxes can even be higher than additional tax revenue from bond returns. In practice, many countries and states exempt interest on government bonds from taxation as documented in Norregaard (1997). In addition to the high cost of administration, administrative difficulties in implementing taxes on international debt holders may also be responsible for such exemptions. This assumption has been commonly used in several pieces of literature, to discuss correlated risks between governments and banks (see, for example, Acharya and Rajan (2013) and Acharya, Drechsler, and Schnabl (2014)) or to discuss sovereign default (see, for example, Cuadra, Sanchez, and Sapriza (2010), Schabert (2010) and Scholl (2017)).

3 Equilibria without policy

I begin the analysis by studying equilibrium with no government guarantee or liquidity regulation. I will first show how the banks form the deposit contract in period 0 and how the tax rate is determined. Based on the deposit contract and the tax rule, depositors choose their withdrawal strategies. In this simultaneous move game, a depositor's strategy is y_i and she aims to maximize her expected utility. I will show that this model generates the self-fulfilling nature of a run, and there always exists a "good" equilibrium in which patient depositors choose to withdraw in period 2. My interest is to study whether a bank run can occur in an equilibrium. A standard approach in the literature to ask under what conditions the following strategy profile is a part of equilibrium:[7]

$$\hat{y}_i(\omega_i, s) = \begin{Bmatrix} \omega_i \\ 0 \end{Bmatrix} \text{ if } s = \begin{Bmatrix} \alpha \\ \beta \end{Bmatrix}, \forall i. \tag{1}$$

In this strategy profile, impatient depositors withdraw in period 1 and patient depositors withdraw in period 2 if the sunspot state is $s = \alpha$, but both types of depositors withdraw in period 1 if the sunspot state is $s = \beta$. I let $s = \beta$ denote the bad state in which a run potentially occurs, and I refer to $s = \alpha$ as the good state. I will say that the financial system is fragile if there exists an equilibrium in which all depositors choose to withdraw in period 1 in $s = \beta$.

Definition 1. The financial system is said to be *fragile* if the strategy profile (1) is a part of an equilibrium; otherwise the financial system is said to be *stable*.

3.1 Deposit contract

A bank forms the deposit contract in which it promises to repay depositors a fixed amount in period 1 and to repay by dividing the remaining resource evenly among the remaining depositors in period 2. Suppose, in period 0, banks expect impatient depositors to always withdraw in period 1 and patient depositors to always withdraw in period 2. Based on this expectation, banks then decide the portfolio structure and payment schedule (c_1, c_2) in period 0. Let x denote the fraction of the total assets placed into project investments; the remaining $(1 - x)$ is invested in government bonds.

In order for the government to be able to repay its debt in the good state, the following condition must be satisfied.

Assumption 1. $d_0 \leq \frac{(1-\pi)(R-R^*)}{R^*}$.

As I show below, this assumption guarantees that investing in the project yields a higher after-tax return at $t = 2$ than holding bonds. In other words, with the tax rate in the good state τ_α, this

[7] See, for example, Cooper and Ross (1998), Peck and Shell (2003), Ennis and Keister (2010) and Li (2017).

assumption guarantees that $R(1-\tau_\alpha) \geq \frac{1}{q_0}$ holds in the no-crisis equilibrium. If this assumption were not satisfied, there would be no equilibrium in which a positive amount of projects could mature, leaving the government zero tax revenue. As a result, anticipating that the government would be unable to repay any bonds in period 2, no agent would purchase government bonds in period 0 and 1. The government would then be unable to raise enough revenue to repay its initial debt and would default on its initial debt in period 0. However, because my interest is to analyze the effects of different policies on financial fragility, I will only focus on cases in which the government can repay its debt in the good state. A sovereign default can be triggered only by a bank run, and does not happen without a run in this model.

Definition 2. There is a *sovereign default* if the government is unable to repay debt fully under an equilibrium strategy profile with $y_i(1,\beta) = 0$.

Bond price: Investors play an important role in determining the price of bonds through their arbitrage between the bonds and the outside option. In period 0, investors and banks expect that the bad state, in which runs occur, will not be realized with probability one, yielding

$$q_0 = \frac{1}{R^*}. \tag{2}$$

The bond prices in periods 0 and period 1 will remain at this level if the good state realizes, meaning

$$q_{1,\alpha} = q_0. \tag{3}$$

Bank's problem: By Assumption 1 and the arbitrage conditions (2) and (3), banks will not liquidate any illiquid assets in the good state, because selling bonds would be more profitable than liquidating projects. Likewise, they will not keep any bond until period 2 in the good state. A bank chooses (c_1, c_2, x) to maximize the expected utility of its depositors such that

$$max_{\{c_1,c_2,x\}}\, \pi u(\,c_1) + (1-\pi)\, u(\,c_2), \tag{4}$$

subject to

$$\begin{aligned}
\pi c_1 &= (1-x)\frac{q_{1,g}}{q_0}, && (5)\\
(1-\pi)c_2 &= xR(1-\tau_\alpha), && (6)\\
c_1 \geq 0, c_2 \geq 0, \quad &\text{and} \quad 0 \leq x \leq 1. && (7)
\end{aligned}$$

The first constraint states that the consumption of impatient depositors in the good state always comes from the returns from bonds. The second constraint states that the bank redeems withdrawals by patient depositors in the good state through returns from projects.

The solution to this problem is characterized by the first-order condition

$$\begin{aligned}
\frac{u'(c_1)}{u'(c_2)} &= R(1-\tau_\alpha)\left(\frac{q_0}{q_{1,g}}\right)\\
&= R(1-\tau_\alpha)\\
&> \frac{1}{q_0}\\
&= R^*.
\end{aligned}$$

The second and last equalities follow from equations (2) and (3), and the inequality is implied by Assumption 1. Notice that $c_2^* > c_1^*$ necessarily holds, because $R^* > 1$. Patient depositors consume more than impatient ones. Under the logarithmic utility function for depositors, the optimal level of project investment is exactly equal to the fraction of patient depositors such as $x^* = 1 - \pi$. The deposit contract is, therefore, given by

$$c_1^* = 1 \ \text{ and } \ c_2^* = R(1-\tau_\alpha). \tag{8}$$

Tax rate: The government levies taxes on matured projects in period 2 to repay the exact amount of outstanding debt. The tax rate in the good state will be

$$\tau_\alpha = \frac{\overbrace{d_0 \frac{1}{q_0}}^{\text{debts}}}{\underbrace{Rx^*}_{\text{tax base}}}. \tag{9}$$

The period-0 bond price in the numerator captures the funding cost for the government. Under Assumption 1, the consumption allocation will be:

$$c_1^* = 1 \text{ and } c_2^* = R - \frac{d_0 R^*}{1-\pi}. \tag{10}$$

3.2 Bank run

Banks suppose that a bank run will not occur when they form the deposit contract, but patient depositors can choose to withdraw in period 1. If this withdrawal happens, banks exhaust their bond holdings and are forced to liquidate projects to redeem the extra withdrawals. Let ℓ be the amount of liquidation needed to accommodate such withdrawals, which increases as more patient depositors withdraw in period 1. Liquidation reduces the number of projects that can mature in period 2, shrinking the tax base. The government must raise the tax rate to be able to repay its debt, and the after-tax return from projects will decrease correspondingly. Let $\tau_\beta \in [0, 1]$ denote the tax rate in the bad state. If there is an equilibrium in which a run occurs and banks have to liquidate projects, τ_β represents the corresponding tax rate.

If this tax rate becomes high enough, banks would find it profitable to liquidate their projects and hold government bonds instead, since the proceeds on these bonds are not taxed. This is true regardless of how many depositors are attempting to withdraw from the bank. To incorporate such incentives, I make the following assumption:[8]

Assumption 2. *Banks liquidate all projects and purchase additional bonds if* $R(1 - \tau_\beta) < rR^*$.

[8]This assumption does not change the deposit contract and does not unnecessarily complicate the analyses. It can be interpreted as "flight to quality" that was observed during the recent crisis. I can, instead, assume that banks do not modify the asset portfolios at all in period 1. Such a modification does not change my results qualitatively.

Note that the return from bonds is $\frac{1}{q_1} = R^*$ which appears on the right hand side of the inequality. This inequality suggests that government bonds yield higher returns than keeping the projects invested despite the bank having to pay liquidation costs. Under this assumption, the maximum tax rate in which the banks keep projects invested is

$$\bar{\tau_\beta} = 1 - r\frac{R^*}{R} < 1. \tag{11}$$

If the tax rate exceeds this threshold, the banks will divert their funds to bonds. The government will lose the tax base and be unable to repay any debts.

Tax channel: In the strategy profile (1), both impatient and patient depositors withdraw in period 1 in $s = \beta$. This is an unexpected event to the banks, in which they must liquidate projects to keep paying c_1^*. The necessary amount of liquidation will be

$$\ell = (1 - \pi)\frac{c_1^*}{r}. \tag{12}$$

The remaining project is $x^* - \ell$, and the bank will run out of assets if $x^* - \ell \leq 0$. Evaluating at the solution to the problem characterized by (4), (5), (6) and (7), banks will always exhaust the assets with this profile because

$$x^* - \ell = (1 - \pi) - (1 - \pi)\frac{1}{r} < 0. \tag{13}$$

The banks liquidate all projects in period 1, the government cannot

$$\tau_\beta = \frac{d_0 \frac{1}{q_0}}{R(x^* - \ell)}.$$

The negative feedback loop: Since $\bar{\tau_\beta} < \tau_\beta$, banks liquidate all projects in period 1 and the government has no tax base on which to levy taxes. The government is unable to fulfill the debt in period 2, which leads to a sovereign default. Investors anticipate this inability to repay and reduce their demand for bonds to zero, meaning that nobody will purchase bonds from banks in period 1, meaning

$$q_{1,\beta} = 0.$$

Banks, then, have to fund all repayments by liquidating projects. The amount of liquidation which banks need to redeem all withdrawals under the sovereign default is

$$\ell_D = \frac{c_1^*}{r} = \frac{1}{r}.$$

The negative feedback loop thus raises the necessary amount of liquidation. The banks will run out all of their assets before period 2 because $x^* - \ell_D < 0$. The banks keep paying c_1^* to redeem withdrawals, and pay nothing once they run out of funds. The probability of a depositor arriving at her bank before the bank runs out of the assets is

$$p = min\left\{\frac{rx^*}{c_1^*}, 1\right\} > 0.$$

A patient depositor, therefore, has a chance to receive positive consumption if she were to withdraw in period 1. However, she receives zero for certain if she were to wait until period 2.

3.3 Equilibria

The above analysis characterizes the deposit contract and the tax rates. It is straightforward to show that the good equilibrium always exists. Recall that an impatient depositor will always strictly prefer to withdraw in period 1 because she values period-1 consumption only, so only the actions of patient depositors need to be considered. Consider the strategy profile $y_i(\omega_i, s) = \omega_i, \forall s, \forall i$, and then a patient depositor always receives $c_2^* > c_1^*$. The associated consumption allocation is the same as (10). As is standard in Diamond and Dybvig (1983), this allocation will be equivalent to the full information efficient allocation.

I now ask whether the strategy profile (1) can indeed be a part an equilibrium and hence whether the financial system is fragile or stable. Based on the self-fulfilling nature of a run, it suffices to show if a patient depositor is incentivized to withdraw in period 1 when $s = \beta$. Suppose

$s = \beta$, banks are unable to repay some of impatient and patient depositors in period 1 because banks run out of funds before repaying everyone as $x^* - \ell_D < 0$. A patient depositor has a chance to receive positive consumption in period 1, but she will receive zero for certain if she waits until period 2. She prefers to withdraw in period 1 over in period 2, and hence I can construct an equilibrium in which depositors follow (1).

Proposition 1. The financial system is always fragile under the no-policy regime

Notice that a sovereign default always occurs in the bad state because the banks liquidate all projects and the government has zero tax revenue.

4 Equilibria with policies

In this section, I study equilibrium outcomes under the three different policy regimes. I derive and compare conditions under which each policy regime is effective in stabilizing the banking system.

4.1 Government guarantees

Suppose that the government guarantees deposits to prevent bank runs in bad times, reassuring the depositors that their banks will repay them. Such guarantees are made through transfers from the government to the banks when banks face the necessity of costly liquidation. By doing so, banks can avoid liquidating projects as long as the government makes the transfer. The government finances this expenditure by issuing additional bonds, and repays these bonds in period 2 together with the bonds issued in period 0.

Deposit contract: The government implements deposit guarantees in bad times, but this scheme does not affect the deposit contract in period 0 because the bad state is not *ex-ante* expected. The contract follows the efficient allocation (10), and there always exists the good equilibrium in which patient depositors withdraw in period 2 because $c_2^* > c_1^*$ holds.

Bank run and guarantees: Consider the strategy profile (1). In $s = \beta$, banks first serve π withdrawals by selling the government bonds, then the government helps banks to redeem extra withdrawals through transfers. The total amount of transfer will be

$$b^{DG} = (1-\pi)c_1^* = (1-\pi). \tag{14}$$

where $(1-\pi)$ is the number of remaining depositors withdrawing after π withdrawals in period 1. To make these transfers, the government issues new bonds in the market and investors may purchase them. The government has to pay an interest rate of $\frac{1}{q_{1,\beta}^{DG}}$ as funding costs. The investors will buy these newly issued bonds if they anticipate the government can fulfill its debt in period 2. The outstanding government debt in period 2 will be

$$d_0 \frac{1}{q_0^{DG}} + b \frac{1}{q_{1,\beta}^{DG}}.$$

In the case that the government can repay all debt in period 2, investors trade the bonds under the arbitrage against the outside option, anticipating the government will pay it back:

$$q_{1,\beta}^{DG} = \frac{1}{R^*}. \tag{15}$$

Correspondingly, the tax rate will be

$$\tau_\beta^{DG} = \frac{d_0 \frac{1}{q_0^{DG}} + b^{DG} \frac{1}{q_{1,\beta}^{DG}}}{Rx^*}. \tag{16}$$

The government is able to guarantee the deposit if the tax rate is below the threshold level (11): $\tau_\beta^{DG} \leq \bar{\tau}_\beta$. While the government finances the amount (14) to serve all extra withdrawals, the banks still have projects as assets and are able to repay more in period 2 than in period 1. A patient depositor is better off deviating to wait until period 2 when $s = \beta$, the strategy profile (1) is not in an equilibrium. Therefore, the deposit guarantee eliminates the equilibrium in which a bank run occurs, given that the government can defray the expenditure.

Crisis equilibrium: The government, however, may not be able to pay the debt back in period 2, meaning that $\tau_\beta^{DG} > \bar{\tau}_\beta$. Investors will then anticipate that the government will default on its debt in period 2, and do not purchase bonds in period 1, meaning

$$q_{1,\beta}^{DG} = 0.$$

In such a case, the government is unable to raise any funds to make a transfer to the banks, and the banks must liquidate projects to serve withdrawal demands in period 1. The banks will exhaust their projects as the liquidation is costly and $\tau_\beta^{DG} > \bar{\tau}_\beta$. The banks repay c_1^* before p withdrawal and 0 otherwise in period 1, and will repay 0 in period 2. A patient depositor prefers to withdraw in period 1 because she has a chance to receive positive consumption by doing so and would receive zero for certain if she were to wait until period 2. I can therefore construct an equilibrium with the strategy profile (1).

Effectiveness: The guarantee can eliminate the crisis equilibrium without distorting the allocation and actual expenditures if it is effective. Its effectiveness, however, depends on whether the government can raise funds or not. In other words, there exists a crisis equilibrium if and only if the government is unable to finance the guarantee, and the ability for the government to finance guarantees is in turn dependent on whether the government can levy sufficient taxes to repay the debts in period 2. The condition for financial stability is then formulated as follows.

Proposition 2. The financial system is $\begin{Bmatrix} fragile \\ stable \end{Bmatrix}$ if $\tau_{2,\beta}^{DG} \begin{Bmatrix} > \\ \leq \end{Bmatrix} \bar{\tau}_\beta$

This proposition implies that once the necessary tax rate exceeds the threshold, the government no longer has a sufficient tax base to fulfill its debt, rendering its guarantee ineffective. By rewriting the condition in Proposition 2 with parameters only, I can derive the condition for the stability of a financial system as

$$d_0 \leq (1-\pi)(\frac{R}{R^*} - 1 - r). \tag{17}$$

The government guarantee becomes more effective in removing the crisis equilibrium as project returns (R) increase and as the initial debt levels decrease (d_0). The intuition behind this condition is that a larger amount of production will expand the tax base for the government, and higher debt levels will minimize leeway for other expenditures. Let $\bar{d_0}^{DG}$ represent the maximum level

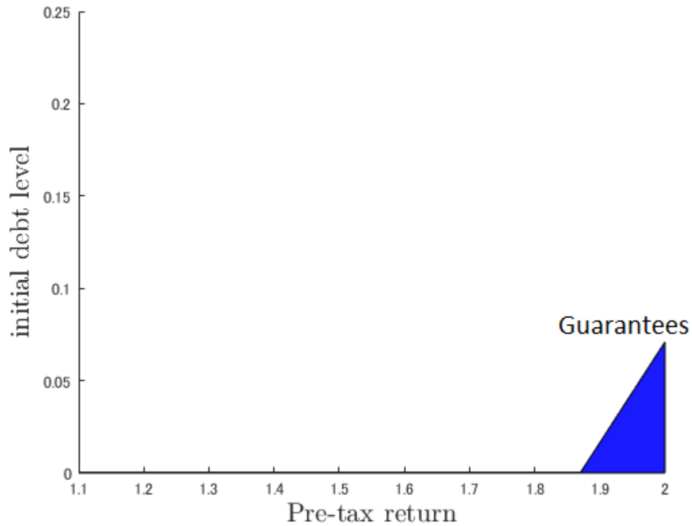

Figure 2: Effectiveness of the guarantee

of initial debt that can be supported in the guarantee regime that satisfies condition (17). The threshold $\bar{d_0}^{DG}$, then, monotonically increases as project returns increase. These results are illustrated in Figure (2), where the colored region represents when the guarantee eliminates the crisis equilibrium. The horizontal axis represents levels of pre-tax returns (R), and the vertical axis corresponds to the initial debt level (d_0). The remaining parameters follow $(r, \pi, R^*) = (0.7, 0.4, 1.1)$.

As for the remaining parameters, outside return (R^*), liquidation value (r) and the fraction of impatient depositors (π) reduce the effectiveness of the guarantee and decrease $\bar{d_0}^{DG}$. Higher outside returns lead to lower bond prices, causing banks to shift their assets from their projects to bonds earlier. Lower bond prices also mean higher funding costs for the government. As a result, the maximum tax rate that the government can implement $(\bar{\tau_\beta})$ will decrease, undermining the effectiveness. The liquidation value affects the maximum tax rate the government can set, because doing so makes it more beneficial for banks to liquidate the projects.

My interest is how this set of parameters compares to the stable sets under the other policy regimes. In the next subsections, I turn to the study of the other policy regimes and derive their conditions for stability.

4.2 Liquidity regulation

The idea of liquidity regulation is to force banks to hold more liquid assets than a set level of liquid assets. An example of such liquidity regulation is the Liquidity Coverage Ratio (LCR) regulation which is newly installed in the Basel III accord. According to this regulation, banks are required to hold enough *high quality liquid assets (HQLA)* to cover their *net cash outflows over the next 30 calendar days (NCOF)* in a stress scenario. The LCR requirement is

$$LCR = \frac{HQLA}{NCOF} \geq 1.$$

Regulation in the model: The LCR regulation would create a buffer for banks to deal with extra withdrawals in period 1 if it binds. In period 0, the banks are required to hold a quantity of government bonds ($HQLA$) equal to their net cash outflows (NCOF), which I take to be a fraction ξ of their total short-term obligations c_1. Given the parameter ξ, a bank chooses (c_1, c_2, x, θ) to maximize the weighted utility of depositors (4) subject to

$$\pi c_1 = \theta \frac{q_{1,g}}{q_0} \tag{18}$$

$$(1-\pi)c_2 = xR(1-\tau_\alpha) + (1-x-\theta)\frac{1}{q_0} \tag{19}$$

$$c_1 \geq 0, c_2 \geq 0, \quad \text{and,} \quad 0 \leq \theta \leq 1-x, 0 \leq x \leq 1, \tag{20}$$

and the LCR constraint

$$\xi c_1 \leq (1-x),$$

where θ represents the amount of the government bonds which the bank sells in period 1. I express the solution to this problem by $(c_1^{LCR}, c_2^{LCR}, x^{LCR}, \theta^{LCR})$. The regulation may reduce the quantity of project investments by banks, leading to higher tax rates. I introduce the upper bound of the regulatory parameter to make sure that the after-tax return from projects is sufficiently large to incentivize banks to invest in projects. The maximum level of regulation $\bar{\xi}$, then, must satisfy

$$R(1-\tau_\alpha(x^{LCR}(\bar{\xi}))) = R^*.$$

A higher ξ than this threshold level raises tax rates and makes investments in long-term projects less profitable than holding government bonds both in period 1 and 2. In such a case, banks will not invest in projects and a necessary tax base will not be assured, rendering the government unable to repay its bonds even in the good state.

The LCR constraint will be slack if $\xi < \pi$, in which case the solution is the same as (10). Otherwise, the deposit contract satisfies

$$\frac{u'(c_1^{LCR})}{u'(c_2^{LCR})} = \frac{\xi R(1-\tau_\alpha^{LCR}) - \xi \frac{1}{q_0^{LCR}} + \pi \frac{1}{q_{1,g}^{LCR}}}{\pi}.$$

Assumption 1 and $\xi < \bar{\xi}$ imply that the government can levy sufficient taxes in period 2 in the good state, and that bond prices will be the same as (2) and (3). Given the logarithmic utility function for $u(\cdot)$, the equilibrium allocation under the LCR regulation will be

$$c_1^{LCR} = \frac{\pi R(1-\tau_\alpha^{LCR})}{\xi R(1-\tau_\alpha^{LCR}) - R^*(\xi - \pi)} \text{ and } c_2^{LCR} = R(1-\tau_\alpha^{LCR}). \tag{21}$$

The amount of project investment and the amount of bonds that banks sell in period 1 will be respectively

$$x^{LCR} = 1 - \frac{\xi \pi R(1-\tau_\alpha^{LCR})}{\xi R(1-\tau_\alpha^{LCR}) - R^*(\xi - \pi)} \text{ and } \theta^{LCR} = \frac{\pi^2 R(1-\tau_\alpha^{LCR})}{\xi R(1-\tau_\alpha^{LCR}) - R^*(\xi - \pi)}. \tag{22}$$

The tax rate is determined by (9), where the amount of project investments (x^{LCR}) depends on the tax rate. While there can be multiple equilibrium tax rates, I suppose that the government chooses the lowest tax rate among those satisfying the rule in order to implement the higher weighted utility of depositors.

Recall that all of c_1^{LCR}, c_2^{LCR} and x^{LCR} are dependent on ξ in addition to τ_α.

Lemma 1. The depositors' utility and the amount of project investments decrease as LCR regulation (ξ) is tightened.

Banks must give up some opportunities to invest in projects by having required liquidity, and hence have less returns to repay. Policy makers choose the lowest ξ if there is more than one ξ capable of making the economy stable, in order to implement the higher weighted utility of depositors.

This allocation implies $c_2^{LCR} > c_1^{LCR}$ for any value of ξ as

$$c_1^{LCR} \leq c_1^* = 1 < R^* < R(1-\tau_\alpha) = c_2^{LCR}.$$

There exists the good equilibrium, and the allocation will be (c_1^{LCR}, c_2^{LCR}).

Run strategy: Under this regulation, banks hold excess liquidity, meaning they hold more liquidity than necessary to repay impatient depositors. I consider the withdrawal strategy (1) in this environment.

The excess liquidity enables banks to redeem extra withdrawals without costly liquidation. The number of depositors which the banks can repay with returns from bonds will be

$$\gamma^{LCR} = \frac{1 - x^{LCR} - \theta^{LCR}}{c_1^{LCR}} = \xi - \pi,$$

where $(1-x^{LCR}-\theta^{LCR})$ shows the amount of excess liquidity. However, banks will eventually run out of the bonds to sell eventually, and they try to redeem remaining withdrawals by liquidation. The amount of necessary liquidation will be

$$\ell^{LCR} = (1 - \pi - \gamma^{LCR})\frac{c_1^{LCR}}{r} = (1-\xi)\frac{c_1^{LCR}}{r}.$$

The bank's ability to redeem these additional withdrawals depends on whether they can pay the contracted repayment to all depositors by liquidation in period 1: $(x^{LCR} - \ell^{LCR})$. The tax rate is determined at

$$\tau_\beta^{LCR} = \frac{d_0 \frac{1}{q_0}}{R(x^{LCR} - \ell^{LCR})}. \tag{23}$$

When banks liquidate all projects, this tax rate will diverge to infinity. However, Assumption 2 implies that the maximum tax rate in which the government can implement is less than positive infinity. Therefore, the condition in which the government does not default is

$$\tau_{\beta}^{LCR} \leq \bar{\tau}_{\beta}, \tag{24}$$

which implies the bank's solvency condition such that

$$x^{LCR} - \ell^{LCR} \geq 0. \tag{25}$$

Suppose that condition (24) holds, and that the government will never default on its debt. In such a case, the bond price will be

$$q_{1,\beta}^{LCR} = \frac{1}{R^*}.$$

The banks will still have a positive amount of assets after serving all withdrawals in period 1. In this situation, a depositor i is better off by deviating from the profile (1), and this strategy profile will not be in an equilibrium.

I now consider the case in which condition (24) does not hold, where the government is unable to repay its debt. In this case, investors anticipate the default and the bond price will be

$$q_{1,\beta}^{LCR} = 0.$$

The necessary liquidation to serve the withdrawals increases as the bonds are worthless as

$$\ell_D^{LCR} = \frac{c_1^{LCR*}}{r}.$$

This suggests that banks must serve all withdrawals by liquidating their projects. The banks will eventually run out funds in period 1 as $(x^{LCR} - \ell_D^{LCR}) << 0$, and will not be able to repay any withdrawals in period 2. Letting $p^{LCR} = min\{\frac{rx^{LCR}}{c_1^{LCR}}, 1\}$ be the probability for a depositor to arrive at her bank before the bank runs out of the assets given the regulation. Similarly to the analyses in the no-policy regime and the guarantee regime, a patient depositor prefers to withdraw in period 1 because she has a chance to receive positive consumption. Therefore, the strategy profile (1) is a part of an equilibrium if and only if the other inequality in the condition (24) holds.

Effectiveness: Liquidity regulation prevents bank runs if it is effective, and its effectiveness in turn depends on whether the excess liquidity is sufficient to avoid the critical level of liquidation. If the necessary liquidation exceeds the critical level, bond prices drop and banks are unable to serve all withdrawal demands through the negative feedback loop. In other words, the tax rate associated with the necessary liquidation should be lower than the threshold tax rate in order for the financial system to be stable. If there exists $\xi \leq \bar{\xi}$ such that the tax rate after necessary liquidation is below the threshold level, policy makers can prevent bank runs through liquidity regulation.

Proposition 3. The financial system is $\begin{Bmatrix} fragile \\ stable \end{Bmatrix}$ if $\tau_{2\beta}^{LCR} \begin{Bmatrix} > \\ \leq \end{Bmatrix} \bar{\tau_\beta}$ for any $\xi \leq \bar{\xi}$.

Note that the equilibrium tax rate in the bad state $\tau_{2,\beta}^{LCR}$ is a function of ξ. The condition for the financial system to be stable can be rewritten as, for any $\xi \leq \bar{\xi}$,

$$d_0 \leq \left(1 - \frac{\xi \lambda R(1-\tau_\alpha)}{\xi R(1-\tau_\alpha) - R^*(\xi - \lambda)}\right)\left(\frac{R - rR^* + \frac{1-\xi}{\xi}}{R^*}\right) - \left(\frac{(1-\xi)(R - rR^*)}{r\xi R^*}\right). \tag{26}$$

where the tax rate (τ_α) is obtained through the tax rule (9) with x^{LCR}. I denote the maximum level of initial debt that can be supported in the liquidity regulation regime that satisfies the equality in condition (26) as $\bar{d_0}^{LCR}$.

Figure 3 depicts this result given the same parameter set as the guarantee regime. The categories labeled on each region indicate which policies can be implemented to eliminate fragility in those economies. For instance, for any economy in the regions labeled as "Liquidity regulation" or "Guarantees or Liquidity regulation", there exists $\xi \leq \bar{\xi}$ such that it satisfies condition (26), meaning that economy can be stabilized by liquidity regulation. In this numerical example, the set of economies that can be stabilized by guarantees is, then, a strict subset of the economies that can be stabilized by liquidity regulation.

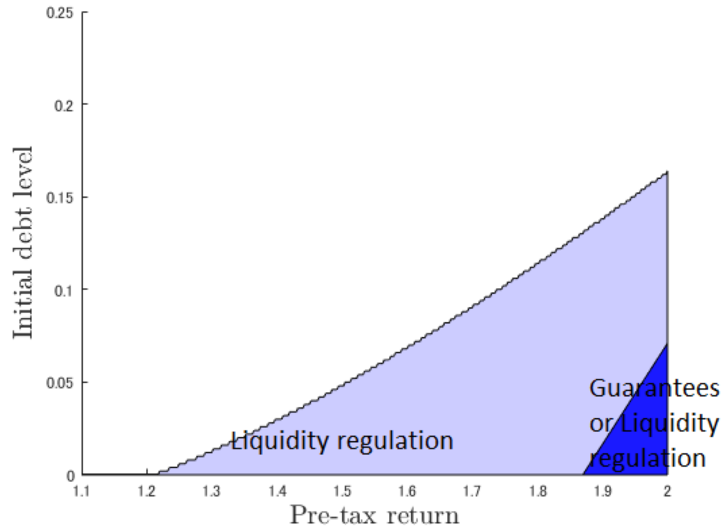

Figure 3: Effectiveness of liquidity regulation

When the return on long-term investment is high and when government's initial debt is small, regulation effectively makes the deposit contract run-proof, requiring banks to have a larger amount of bonds and to reduce expected short-term outflows. The value of these bonds depends on whether the government can repay them. Having higher returns on investment and lower initial debt helps government repay the bonds, and banks can redeem some extra withdrawals from returns from the bonds without liquidating long-term projects, as $\bar{d}_0^{LCR}$ increases as R increases.

In the next subsection, I turn to the study of the combination of these two policies, and discuss how they interact with each other.

4.3 Policy mix

Policy makers may consider adopting liquidity regulation and government guarantees together. Banks would be required to hold some level of liquid assets according to the regulation, and the government would make transfers to the banks once they deplete their liquid assets. One difference that this has from adopting liquidity regulation alone is, therefore, whether the government prevents the banks from liquidating any single project or not after exhausting their liquid assets.

Deposit contract: In period 0, banks face an identical problem to the liquidity regulation regime because, like in the liquidity regulation regime, the bad state is not expected to occur. The deposit contract will be the same as the one in the liquidity regulation regime (21-22). There exists the good equilibrium because withdrawing in period 2 gives patient depositors $c_2^{LCR} > c_1^{LCR}$.

Run strategy: I now consider the strategy profile (1). Suppose $s = \beta$, banks would still have some government bonds to serve further withdrawals after serving π withdrawals, and can redeem withdrawals without any liquidation for up to γ^{LCR} withdrawals, analogous to the liquidity regulation regime. Once the banks exhaust the government bonds, the government begins to make transfers in order to avoid liquidation. The necessary amount to prevent liquidation is

$$b^{MIX} = (1 - \xi)c_1^{LCR}.$$

Notice that this necessary amount of transfers is smaller than b^{DG}. This is not only because $(\pi - \xi)$ depositors are served through the excess liquidity, but also because banks make a safer deposit contract under liquidity regulation ($c_1^{LCR} < c_1^*$). Thus, liquidity regulation diminishes the necessary amount of transfers per depositor in the guarantee. Banks have to give up some projects in holding more bonds, while it will be less costly for the government to rescue the banks.

However, it is unclear whether the tax rate would be lower than that of the guarantee regime, because regulation reduces the tax base. Suppose that the government has a sufficient tax base so that the government could repay in period 2, and then the tax rate will be:

$$\tau_\beta^{MIX} = \frac{d_0 \frac{1}{q_0^{MIX}} + b^{MIX} \frac{1}{q_{1,\beta}^{MIX}}}{Rx^{LCR}}. \tag{27}$$

The bond price in the bad state is

$$q_{1,\beta}^{MIX} = \frac{1}{R^*}. \tag{28}$$

In this case, the government can guarantee all withdrawals after ξ withdrawals and $\tau_\beta^{MIX} \leq \bar{\tau}_\beta$. Similarly to the guarantee regime, patient depositors are better off deviating from the strategy profile (1) in order to receive the leftovers. The combination of policies, then, eliminates the crisis equilibrium if the government can raise funds for the transfer.

Crisis equilibrium: The government may not be able to implement a tax rate below the threshold, resulting in $\tau_{\beta}^{MIX} > \bar{\tau}_{\beta}$. Anticipating the default, investors will not purchase bonds from banks, and hence

$$q_{1,\beta}^{MIX} = 0.$$

Banks need to liquidate projects to redeem withdrawals in period 1. As in the liquidity regulation regime, the banks eventually run out of projects to liquidate in period 1. A patient depositor will consume zero if she waits until period 2, otherwise she has a chance of consuming positive amounts. I can then construct an equilibrium with the strategy profile (1), in which patient depositors will bank run in the bad state.

Effectiveness: Liquidity regulation constrains banks to invest less in projects, resulting in a reduction of the tax base. However, it reduces the fiscal cost to guarantee deposits. The policy combination can, therefore, decrease the contingent liability on the bad state by sacrificing the tax base. The policy combination, then, may or may not complement the government guarantee, and its effectiveness depends on which of a decrease in the contingent liability or a decrease in the tax base is more significant.

Proposition 4. The financial system is $\begin{Bmatrix} fragile \\ stable \end{Bmatrix}$ if $\tau_{2,\beta}^{MIX} \begin{Bmatrix} > \\ \leq \end{Bmatrix} \bar{\tau}_{\beta}$ for any $\xi \leq \bar{\xi}$

where the equilibrium tax rate in the bad state $\tau_{2,\beta}^{MIX}$ is a function of ξ. Note that this proposition is equivalent to Proposition 2 if $\xi \leq \pi$, because the liquidity regulation does not constrain the bank's behavior. This condition for the financial system to be stable can be rewritten as; for any $\xi \leq \bar{\xi}$,

$$d_0 \leq \left(1 - \frac{\xi\lambda R(1-\tau_\alpha)}{\xi R(1-\tau_\alpha) - R^*(\xi - \lambda)}\right)\left(\frac{R}{R^*} - r + \frac{1-\xi}{\xi}\right) - \left(\frac{1-\xi}{\xi}\right). \tag{29}$$

where the tax rate (τ_α) is obtained through the tax rule (9) with x^{LCR}. Let $\bar{d_0}^{MIX}$ satisfy the equality in condition (29) and represent the maximum level of initial debt that can be supported in the policy mix regime. The set of economies that can be stabilized by guarantees is a strict

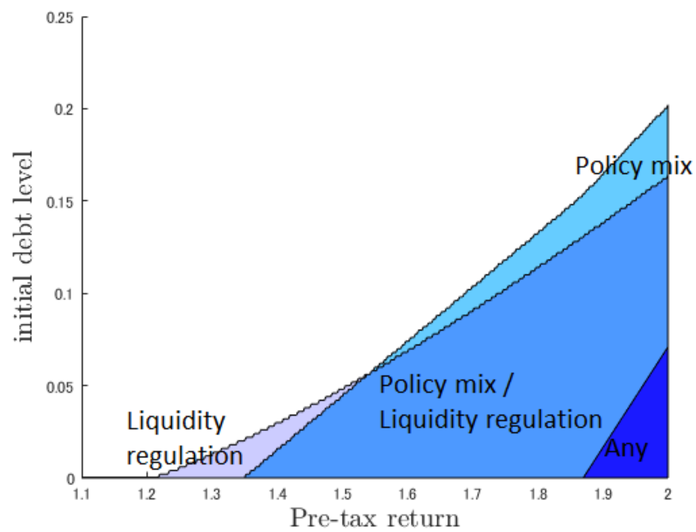

Figure 4: Effectiveness of the combination of the two policies

subset of the economies that can be stabilized by a policy mix.

Lemma 2. *For any parameter sets,* $\bar{d_0}^{MIX} \geq \bar{d_0}^{DG}$.

This result is illustrated in Figure 4. For any economy in the region labeled as "Policy mix" or "Any", there exists $\xi \leq \bar{\xi}$ satisfying condition (29), and that economy can eliminate fragility by combining the guarantee and liquidity regulation.

While these two policies may be lacking individually, in certain situations, they may complement each other in a way that makes them effective when used in combination. Specifically, liquidity regulation lowers the banks' short-term liabilities, decreasing government liabilities in the event of a run. As project returns increase, and as initial debt levels decrease, weaker regulation can eliminate fragility. Higher project returns and smaller initial debt levels give the government more room to guarantee deposits. As regulation continues to grow weaker, it will eventually become slack, and the economy will be able to eliminate fragility solely by the guarantee.

4.4 Discussion

The negative feedback loop between the banking sector and the government limits the effectiveness of government guarantees. Government guarantees are the government's contingent liability on the soundness of the banking system. If investors anticipate that the government cannot repay the debts, it cannot raise funding for guarantees, which makes the guarantee policy ineffective. Government guarantees are effective if condition (17) is satisfied, for instance, in economies with high returns and low debt, but will be ineffective if the return decreases or debt increases. I have demonstrated that the government guarantees may be complemented by liquidity regulation and that the combination of these two policies is needed to prevent a run in some cases. The effectiveness of liquidity regulation is also limited by the negative feedback loop. Restricting the bank's maturity transformation has the potential to eliminate the source of fragility. However, restricting the banks' investments in projects hurts the tax revenue. If the tax base shrinks substantially, the government will be unable to repay the debts and defaults on the debts. The tightness of regulations is, therefore, limited by the possibility of sovereign defaults, and hence liquidity regulation is not effective in eliminating fragility in some cases. It is not trivial whether guarantees and regulations complement each other. In some cases, a combination of the two policies is needed to prevent a run. In other cases, liquidity regulation alone is effective and adding guarantees would make the financial system fragile.

Proposition 5. $\bar{d}_0^{LCR} \begin{Bmatrix} \geq \\ < \end{Bmatrix} \bar{d}_0^{MIX}$ as $\frac{R}{2} \begin{Bmatrix} \leq \\ > \end{Bmatrix} rR^*$.

The difference between the combination of two policies and liquidity regulation alone is how to redeem deposits after π withdrawals in period 1. In the policy mix, the government transfer funds to the banks and prevents any liquidation. This action increases the government's liability, while the tax base does not shrink. In liquidity regulation alone, the banks start liquidating projects. The government does not have extra expenditures, but the liquidation decreases the tax base. When issuing new debts is cheaper or when a project yields a higher return, the policy mix is more likely to eliminate fragility than liquidity regulation alone. On the other hand, when liquidating projects are more costly, the liquidity regulation alone is more likely to eliminate fragility. These results imply that the two policies may complement each other in the economy with high returns, low government funding costs, or low liquidation costs.

Regulations, however, entail welfare loss because it reduces the amounts of the bank's repayment. From a welfare perspective, guarantees will be the most preferable if condition (17) is satisfied. Otherwise, a policy regime that can get rid of fragility with a weaker regulation implements better welfare.[9] Proposition 6 establishes that an effective policy regime with a lower fiscal cost just needs a weaker regulation.

Proposition 6. Suppose $d_0 < min(\bar{d_0}^{LCR}, \bar{d_0}^{MIX})$ is satisfied, then

$$\begin{Bmatrix} \text{Liquidity regulation} \\ \text{Policy mix} \end{Bmatrix} \text{ implements higher depositors' utility if } \frac{R}{2} \begin{Bmatrix} \leq \\ > \end{Bmatrix} rR^*.$$

Numerical example: These results can be discussed through Figure 4 in which an economy in a colored region satisfies at least one of the conditions to be stable. All of the three policy regimes are effective in economies with high returns and low debt. If the return decreases or the debt level increases, guarantees becomes ineffective whereas liquidity regulation and the policy mix remain effective. In economies with high returns and high debt levels, only the policy mix is effective. Conversely, liquidity regulation is the only solution to eliminate fragility in economies with low returns and low debt levels. The policy mix can support higher levels of initial debt if the return is high, otherwise liquidity regulation can sustain higher levels of initial debt. At the point where the boundaries of the liquidity regulation regime and the policy mix regime cross, these two regimes need the same level of regulation to be effective and achieve the same consumption allocation. An economy outside the colored region cannot be stable by any of the policies. Such an economy either has multiple equilibria or does not have any equilibrium in which the government is able to repay its debt.

5 Concluding remarks

I have studied the effectiveness of government guarantees, liquidity regulation and a combination of these two policies in stabilizing the banking system given the negative feedback loop between banks and the government. To evaluate the fiscal costs of these policies, I have extended the model of Diamond and Dybvig (1983) to include a government that issues and may default on

[9] Recall that I have assumed a sufficiently large default cost (δ) such that a fragile financial system is strictly worse than a stable one in a welfare perspective.

its debt. Additionally, my model has three linkages between banks and government: tax revenue, guarantees and government bond prices. I have found that an economy is always fragile under the no-policy regime, and that policies are not always effective in eliminating fragility. Both guarantees and liquidity regulation have negative effects on debt sustainability, either through expenditures or tax revenue. The effectiveness of the policies is restricted by debt sustainability, and an ineffective policy will result in a banking crisis with sovereign default.

I have derived the conditions for each policy to be effective in eliminating fragility, and have shown that liquidity regulation complements guarantees by reducing the fiscal costs of the guarantee. This is because regulation requires banks to hold excess liquidity and decrease the banks' short-term liability. When guarantees are ineffective, either liquidity regulation alone or a combination of these two policies may be most effective depending on their costs, in particular, the government's funding costs to serve a depositor and the losses of the tax base associated with a run. Regulation is more likely to be effective than the policy mix in economies with low liquidation costs, high funding costs and low project returns. In contrast, the policy mix is more likely to be effective in economies with high liquidation costs, low funding costs and high project returns. In other words, the two policies are more likely to complement each other in these cases.

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A Appendix: Proofs for selected results

Lemma 1. (i) Amount of project investment

Recall the optimal level of project invested is determined at Equation (22). I must show $\frac{\partial x^{LCR}}{\partial \xi} < 0$ where the tax rate τ_α is determined at the fixed point in Equation (9).

$$\begin{aligned}
\frac{\partial x^{LCR}}{\partial \xi} &= -\frac{\pi R(1-\tau_\alpha)}{\xi R(1-\tau_\alpha) - R^*(\xi - \pi)} + \xi \pi R(1-\tau_\alpha) \frac{R(1-\tau_\alpha) - R^*}{(\xi R(1-\tau_\alpha) - R^*(\xi-\pi))^2} \\
&= -\frac{\pi^2 R^* R(1-\tau_\alpha)}{(\xi R(1-\tau_\alpha) - R^*(\xi-\pi))^2} \\
&< 0
\end{aligned}$$

Despite τ_α changes upon ξ, the last inequality always holds because $\tau_\alpha \in [0,1]$.

(ii) Depositors' utility

$\frac{\partial x^{LCR}}{\partial \xi} < 0$ implies that the tax rate must increase to compensate the shrink of the tax base, leading to $\frac{\partial c_2^{LCR}}{\partial \xi} < 0$. Additionally,

$$\begin{aligned}
\frac{\partial c_1^{LCR}}{\partial \xi} &= -(R(1-\tau_\alpha) - R^*) \frac{\pi R(1-\tau_\alpha)}{(\xi R(1-\tau_\alpha) - R^*(\xi-\pi))^2} \\
&< 0
\end{aligned}$$

where $(R(1-\tau_\alpha) - R^*) > 0$ is implied by Assumption 1 and $\xi < /bar\xi$. Both impatient and patient depositors are therefore worse off by the regulation. □

Proposition 5. Let $\bar{d_0}^{LCR}$ and $\bar{d_0}^{MIX}$ be the maximum levels of initial debts in which a stable economy can accommodate in the liquidity regulation regime and the policy mix regime respectively. Equation (26) and (29) determine these threshold levels when their equalities hold.

The subtraction gives;

$$\bar{d_0}^{MIX} - \bar{d_0}^{LCR} = \frac{R - rR^*}{r} - R^* \tag{30}$$

The liquidity regulation regime could thus tolerate more initial debts than the policy mix regime if $\frac{R-rR^*}{r} - R^* > 0$, or $\frac{R}{2} > rR^*$, and vice versa. $\square$

Proposition 6. Suppose a set of parameters satisfies $\frac{R}{2} > rR^*$. Let $\tilde{d_0} = \bar{d_0}^{LCR}$, then $\bar{d_0}^{MIX} > \tilde{d_0}$ by Proposition 5. Since $\frac{\partial \bar{d_0}^{MIX}}{\partial \xi} > 0$ and $\frac{\partial \bar{d_0}^{LCR}}{\partial \xi} > 0 \; \forall \xi \in [\pi, \bar{\xi}]$, I get $\xi^{MIX} < \xi^{LCR}$. Lemma 1 implies $(c_1^{MIX}, c^{MIX_2}) > (c_1^{LCR}, c^{LCR_2})$. An analogy can be applied to the other inequality. $\square$