Wesleyan Economic Working Papers

http://repec.wesleyan.edu/
N°: 2024-006

Rapid Bank Runs and Delayed Policy Responses

Ryuichiro Izumi & Yang Li

June 2024

WESLEYAN UNIVERSITY

Department of Economics
Public Affairs Center
238 Church Street
Middletown, CT 06459-007

Tel: (860) 685-2340
Fax: (860) 685-2301
http://www.wesleyan.edu/econ

Rapid Bank Runs and Delayed Policy Responses *

Ryuichiro Izumi†
Wesleyan University
rizumi@wesleyan.edu

Yang Li‡
Singapore University of Social Sciences
liyang@suss.edu.sg

June 14, 2024

Abstract

The banking turmoil of 2023 highlighted how technological advancements have significantly accelerated the speed of bank runs. This paper investigates the impact of these faster bank runs on the effectiveness of policy interventions by interpreting them as a constraint on the relative speed of policy responses. Using a model of bank runs and ex-post policy responses, we examine how delays caused by this constraint affect financial fragility and welfare. We find that while delays exacerbate welfare loss by distorting allocations, they may also decrease fragility by making banks more cautious. We explore the optimal level of structural delay, balancing the trade-off between distributional distortions and financial fragility.

Keywords: Bank runs; Delayed Intervention; Speed of Bank Runs.

JEL classification: G21, G28, E58

*We thank Todd Keister for his comments and suggestions. Spencer Lee provided excellent research assistance. All errors are ours.

†Corresponding author. 238 Church Street, Middletown, CT, 06457, U.S.A.

‡463 Clementi Road, 599494, Singapore.

1 Introduction

The banking turmoil in 2023 highlighted the rapid withdrawals that can occur during bank runs. Rose (2023) documents that daily withdrawals at troubled banks during this period were substantially larger than past bank runs. For instance, Silicon Valley Bank and Signature Bank each lost 20-25% of their total deposits within a single day. In contrast, during the 2007-8 financial crisis, Washington Mutual experienced withdrawals amounting to only 10.1% of total deposits over 16 days, and Wachovia saw only 4.4% withdrawn over 19 days. However, the accelerated speed of these bank runs may not be limited to the 2023 turmoil. It might be a common feature of bank runs in an era of digitization. Both Rose (2023) and Bowman (2024) attribute the increased speed of bank runs to technological advancements in banking, such as electronic withdrawal technologies and rapid communication among depositors. These advancements may cause faster withdrawals in future bank runs again, necessitating the incorporation of the speed of bank runs into our understanding of financial stability and policy measures. In particular, a faster bank run poses a significant challenge for policymakers, as it requires swift responses that may undermine the effectiveness of policy interventions. To fully understand the impact of accelerated bank runs on policy effectiveness, it is essential to systematically analyze how the evolving environment influences the actions of banks, depositors, and policymakers.

This paper integrates the concept of the speed of bank runs into a model of bank runs and policy responses. Specifically, we interpret a faster bank run as a limitation on the relative speed of policy intervention. As bank runs accelerate, the ability of policymakers to choose the timing of intervention becomes constrained, resulting in inevitable delays. Our primary interest lies in examining how this restriction influences financial fragility and welfare. Specifically, we study a version of the Diamond and Dybvig (1983) model with limited commitment to study a restriction on the timing of ex-post policy responses, such as a deposit freeze. Following Ennis and Keister (2009, 2010), a policymaker can suspend banks' repayments in times of runs but lacks commitment, and the timing of intervention is chosen ex-post. While a policymaker optimally desires to intervene as soon as a run is identified, we assume they face *a structural delay* imposed by an exogenous constraint. This constraint, which we later endogenize, represents the earliest point at which a policymaker can suspend payments. Such a constraint has an obvious negative effect: the size of bank runs becomes larger as the bank must continue making repayments by liquidating assets. In fact, we find that, given the probability of a bank run, a delay due to this additional constraint always worsens welfare.

However, the restriction on a quicker response may have a nontrivial effect on fragility. The restriction is publicly known, influencing not only the policymaker's choice of suspension point but also banks' behavior. When banks anticipate a delay in suspensions, they begin to act more cautiously in response to the possibility of a bank run. Specifically, they reduce the amount of short-term payments, leading to an interesting result in this paper. On the one hand, a delayed intervention caused by the restriction incentivizes a depositor to run on the bank because fewer resources will be left after a suspension. On the other hand, anticipating a delayed intervention, banks pay less in the short-term and retain more resources after the suspension, which mitigates the run incentive. We find that the net effect is not obvious, and we provide the conditions under which a delayed intervention can decrease fragility. This result suggests that as bank runs become faster, the banking system may become less fragile because banks become more cautious about runs. A tighter restriction on the relative speed of policy responses may cause two competing effects: distributional distortions and decreased fragility.

To explore the competing effects, we present the comparative statics of changing the degree of restriction. By internalizing the effect on fragility, we re-examine the welfare implications of the restriction on the timing of policy responses. In this exercise, we use parameter sets motivated by real-world contexts to examine the empirical plausibility of our mechanism. We find that, although there is a mechanism where a delayed intervention may decrease fragility, this is not a plausible channel in the current economy. This exercise implies that a faster bank run worsens welfare through both the distributional distortions and increased fragility in the current economy.

Finally, we consider the possibility that policymakers can adjust the degree of restriction ex-ante, subject to an adjustment cost. For example, policymakers could invest in surveillance and monitoring facilities to respond more quickly in times of crisis, though such investments are costly. Considering these costs, we show that it may be optimal to permit a structural delay of suspension, suggesting that policymakers may not need to invest sufficiently to respond promptly during a crisis.

Related literature: Our approach contributes to the discussion on self-fulfilling bank runs by considering their speed in the context of policy responses. Since the seminal work of Bryant (1980) and Diamond and Dybvig (1983), the literature has evolved to study the causes and amplification mechanisms of bank runs. A major specification is sequential service as in Wallace (1988) and Wallace (1990). A policymaker learns whether a run is underway or not over the flow of withdrawals and is allowed to suspend payments anytime. Ennis and Keister (2009) and Ennis and Keister (2010) consider that a policymaker chooses a suspension timing ex-post optimally, and this specification has been widely used to study ex-post policy responses in the literature.[1] The common assumption here is that a policymaker can respond immediately when a run is identified. This paper assumes that a policymaker *cannot* respond immediately even if a bank run is identified, and this restriction is modeled as a constraint on the choice of the timing of policy responses. Doing so allowed us to find the novel result that such a restriction may decrease fragility by making banks more cautious.

This paper further contributes to a broader discussion on ex-post policy interventions. A wide range of policy instruments have been considered in the literature, including bail-outs (Philippon & Skreta, 2012; Bruche & Llobet, 2014; Keister, 2016; Faria-e-Castro, Martinez, & Philippon, 2017; Segura & Suarez, 2019), bank closure and forbearance (Mailath & Mester, 1994; Schilling, in press, Acharya & Dreyfus, 1989; Allen & Saunders, 1993; Morrison & White, 2013), and bail-ins (Walther, 2016; Keister & Mitkov, 2024). Koenig and Mayer (2022) study the optimal timing of intervention in a model where the uncertainty about bank solvency is gradually discerned, considering liquidity support. Both Ennis and Keister (2009) and Koenig and Mayer (2022) showed that a policymaker may optimally delay ex-post interventions. Our paper has a different mechanism behind a delayed

[1] See, for example, Keister (2016), Keister and Narasiman (2016), Gao and Reed (n.d.), and many others.

intervention, caused by an ex-ante incentive of a policymaker. In our framework, the ex-post incentive of a policymaker is to intervene as soon as a run is identified as in Ennis and Keister (2010), but the earliest timing of intervention is restricted. The policymaker can choose the degree of this restriction ex-ante, and hence, if it is optimal, the delayed intervention is caused by the policymaker's ex-ante incentive. Such a restriction influences banks' behavior, which is the key channel that motivates the policymaker to delay the intervention.

2 The model

Our model builds on Ennis and Keister (2010), which is a version of the Diamond and Dybvig (1983) model augmented to include limited commitment friction. A policymaker intervenes in the banking system ex-post optimally, but we add a restriction on when the policymaker can intervene. This section describes the model environment, including agents and technologies, and then defines financial fragility in this environment.

2.1 The environment

We consider an economy with three periods indexed by $t = 0, 1, 2$. The economy is populated by a $[0, 1]$ continuum of ex-ante identical depositors, indexed by i. We suppose that each depositor has preferences of the following CRRA form:

$$u\left(c_1, c_2; \omega_i\right) = \frac{\left(c_1 + \omega_i c_2\right)^{1-\gamma}}{1-\gamma},$$

where c_t represents consumption of a single good in period t and the coefficient of relative risk-aversion γ is assumed to be greater than one. The parameter ω_i is a binomial random variable with support $\Omega \equiv \{0, 1\}$ and represents a depositor's type. If $\omega_i = 1$, depositor i is patient, while she is impatient if $\omega_i = 0$. A depositor's type is revealed in period 1 and privately observed by each depositor. Each depositor is chosen to be impatient with a known probability $\pi \in (0, 1)$, and the fraction of impatient depositors in each location equals π.

Technologies: Each depositor is endowed with one unit of the good at the beginning of period 0. There is a single, constant-returns-to-scale technology for transforming this endowment into consumption in the later periods. Specifically, a unit of the good invested in period 0 yields $R > 1$ units in period 2 but only one unit in period 1.

Financial intermediation: The investment technologies are operated at a central location, where depositors pool and invest resources together in period 0 to insure individual preference risk. This intermediation technology can be interpreted as a financial intermediary or a *bank*.[2] In period 1, upon learning her preference type, each depositor chooses either to withdraw her funds in period 1 or to wait until period 2. Those depositors who contact the bank in period 1 arrive one at a time in the order given by their index i.[3] This index is private information, and the bank only observes that a depositor has arrived to withdraw. Under this sequential service constraint, as in Wallace (1988, 1990), the bank determines the payment to each withdrawing depositor based on the number of withdrawals that have been made so far. The objective of the bank is to maximize the expected utility of depositors

$$\mathcal{W} = \int_0^1 E\left[u\left(c_1(i), c_2(i); \omega_i\right)\right] di.$$

Policymaker: A benevolent policymaker can intervene in the banking system ex-post in the form of deposit freezes. Once the policymaker intervenes in the bank, the policymaker takes over the bank's operation and allocates the remaining resources efficiently.[4] However, the policymaker can intervene only after $\theta \in [\pi, 1]$ withdrawals in period 1, which is publicly known. A higher θ implies that the policymaker will require a longer time to intervene.[5]

Limited commitment: The bank and the policymaker cannot pre-commit to their future

[2]As is standard in the literature, the bank can be interpreted as a coalition of depositors or a representative bank. Either interpretation leads to the same results in this paper.

[3]This specification implies that a depositor knows her position in the order of withdrawals before making her withdrawal decision. This assumption is commonly used in the literature. See, for example, Green and Lin (2003) and Ennis and Keister (2010).

[4]This intervention can be interpreted as the bank entering into resolution.

[5]The main focus of this paper is how the values of θ affect fragility, and thus, we will first study comparative statics of θ. Section X endogenizes the value of θ.

actions. While Diamond and Dybvig (1983) show that a deposit freeze with commitment can eliminate a bank run equilibrium, Ennis and Keister (2009) find the time-inconsistency of such an intervention. In this paper, we prohibit the bank and policymaker from using any time-inconsistent policy so that a bank run arises as an equilibrium phenomenon. Thus, after π withdrawals, the policymaker learns whether a run is underway or not: A bank run occurs if and only if an extra withdrawal is requested. However, the policymaker cannot immediately suspend the payments. Only after θ withdrawals can the policymaker suspend the payments and take over the bank's operation. Once the bank is placed into resolution, the policymaker allocates the remaining resources efficiently, which gives some goods to the remaining impatient depositors in period 1 and the rest of the funds to the remaining patient depositors in period 2.[6]

2.2 Financial crises

We follow Peck and Shell (2003) and many others in introducing the probability of bank runs through an extrinsic *sunspot* variable. The economy will be in one of two sunspot states, $s \in S \equiv \{\alpha, \beta\}$ with probabilities $\{1-q, q\}$.[7] Depositors observe the realization of the sunspot variable at the beginning of period 1 and may condition their withdrawal strategies on the sunspot variable, while the banks do not observe the sunspot state and must infer it based on the observed withdrawal behavior. In period 1, each depositor chooses to withdraw either in period 1 or 2 based on the sunspot variable and her preference type:

$$y_i : \Omega \times S \to \{0, 1\},$$

where $y_i = 0$ corresponds to withdrawing at $t = 1$ and $y_i = 1$ corresponds to withdrawing at $t = 2$. Let y denote a profile of withdrawal strategies for all depositors. An impatient depositor will choose to withdraw at date 1 in both states, since she does not value consumption in period 2.

[6] A bank run thus stops as soon as the policymaker steps in, and the policymaker can distinguish impatient depositors and patient depositors as discussed in Ennis and Keister (2010).

[7] The value of q is constant and can take any values in $[0, 1]$. We will later study what value of q makes the strategy profile in interest be part of equilibrium.

A patient depositor may or may not withdraw in period 1, and we say that a bank run occurs if a positive measure of patient depositors withdraws in period 1. Therefore, the banks infer that a run is underway if withdrawals continue after the first π withdrawals. We assume that the banks then react and that the run stops, as in Ennis and Keister (2009) and many others.[8] In view of this discussion, we consider the following *run strategy profile* for a depositor i:

$$\begin{aligned} y_i(\omega_i, \alpha) &= \omega_i \qquad\qquad \text{for all } i, \text{ and} \\ y_i(\omega_i, \beta) &= \left\{ \begin{array}{c} 0 \\ \omega_i \end{array} \right\} \text{for} \left\{ \begin{array}{c} i \leq \theta \\ i > \theta \end{array} \right\}. \end{aligned} \tag{1}$$

Under this profile, state α has no run while state β incurs a run: each patient depositor with $i \leq \theta$ chooses to withdraw early in state β. The banks do not know the realization of the sunspot variable and cannot initially infer if a run is underway. After a fraction θ of depositors has been served, the policymaker can respond. Upon intervention, all remaining impatient depositors are served in period 1, while all remaining patient depositors are directed to withdraw in period 2. The following definition provides the notion of financial fragility that we use in this paper.

Definition 1 *A banking system is said to be fragile if the strategy profile* (1) *is part of an equilibrium; otherwise, the banking system is said to be stable.*

Our focus is to study the set of values for q that makes the run strategy profile an equilibrium and, hence, the bank fragile. If q is small, the bank may offer early repayments that are large enough to incentivize depositors to run on the banks if they anticipate others will do so. Then, the run strategy profile constitutes an equilibrium. If q is sufficiently large, however, the banks become conservative in making early payments. When q is large enough, depositors no longer have an incentive to run on the banks, and the banks are stable. The next section establishes the set of

[8] This can be interpreted as the banks moving into a resolution regime after θ withdrawals. For example, the court involves this resolution scheme and can distinguish impatient from patient depositors, allowing only the remaining impatient depositors to withdraw in period 1. This assumption can be generalized by introducing a richer space for the sunspot variable in which runs can occur in multiple waves. Having multiple waves of runs, however, does not change the mechanisms we will present, and the results will remain qualitatively unchanged. See, for example, Ennis and Keister (2009, 2010).

q that is consistent with fragile banking systems.

2.3 Timeline

The timing of events is summarized as follows. In period 0, depositors place their endowments in the banks, and the period ends. At the beginning of period 1, depositor i learns her type ω_i and the realized sunspot state, and can choose either to withdraw in period 1 or wait until period 2. At the same time, the banks choose a repayment plan, and then begin repaying depositors sequentially. After π withdrawals, the policymaker can infer the sunspot state by observing whether additional withdrawals occur or not. If a run does not happen, withdrawals stop after π withdrawals, and the patient depositors will get repaid in period 2. If a run is underway, withdrawals continue after π withdrawals, and the policymaker can suspend payments after θ withdrawals as a resolution. The remaining impatient depositors receive payments in period 1, and the remaining patient depositors receive payments in period 2.

2.4 Discussion

Our setup for policy interventions is reminiscent of the banking turmoil in early 2023. First, the policymaker cannot take action until θ withdrawals, although they can infer whether a run is underway or not at π withdrawals. Thus, by assumption, the policy response is inevitably delayed in this setup, and we interpret the difference $(\theta - \pi)$ as a structural delay in the response. Bank runs at the Silicon Valley Bank and a couple of other banks were extraordinarily fast, and Rose (2023) discusses changes in technologies surrounding withdrawals, such as online banking and the role of social networks as the driving forces. A fast bank run makes a swift policy response challenging, and responses may necessarily be delayed. Our setup incorporates this observation into the model to study its implication for fragility. Second, we assume that the bank cannot respond to an occurrence of bank runs by rescheduling its payment before the policymaker comes. This assumption can be interpreted either as (i) the bank cannot react quicker than the policymaker or (ii) the bank keeps paying an obligation per withdrawal request (e.g., ATM). In either case, this assumption is consistent with the observation that the troubled banks kept paying their obligations until the FDIC stepped in.

3 Equilibrium

We study a simultaneous-move game played by depositors and the bank, where the policymaker is not a strategic player in this game. As it becomes clear later, the policymaker always intervenes as soon as θ withdrawals are made in case of a crisis. Anticipating this policy response, the bank chooses the payment schedule, and the depositors choose when to withdraw. We suppose that each depositor chooses the run strategy profile and we study how the bank responds to it. By doing so, we are interested in finding a set of q that allows the profile to constitute an equilibrium. When the depositors play the run strategy profile, the bank solves the following problem.

$$\max_{\{c_1, c_{1\beta}, c_{2\alpha}, c_{2\beta}\}} (1-q)\left\{\pi u(c_1) + (1-\pi)u(c_{2\alpha})\right\} + q\left\{\theta u(c_1) + (1-\theta)\left[\pi u(c_{1\beta}) + (1-\pi)u(c_{2\beta})\right]\right\}, \quad (2)$$

subject to

$$\pi c_1 + (1-\pi)\frac{c_{2\alpha}}{R} = 1, \quad (3)$$

$$\theta c_1 + \pi(1-\theta)c_{1\beta} + (1-\pi)(1-\theta)\frac{c_{2\beta}}{R} = 1. \quad (4)$$

The first constraint corresponds to state α, where period-1 withdrawals stop at π. The bank keeps paying c_1 in period 1 and will pay $c_{2\alpha}$ to each patient depositor in period 2. The second constraint corresponds to state β, where withdrawals continue until the policymaker steps in at θ withdrawals. The measure of the remaining impatient depositors is $\pi(1-\theta)$, who receive $c_{1\beta}$ in period 1. On the other hand, the measure of the remaining patient depositors is $(1-\pi)(1-\theta)$, and they receive $c_{2\beta}$ in period 2. Let η_α and η_β denote the Lagrangian multiplier on the first and second constraints, respectively. Then, we obtain the following first-order conditions:

$$
\begin{aligned}
((1-q)\pi + q\theta)\, u'(c_1) &= \pi\eta_\alpha + \theta\eta_\beta = \pi(1-q)Ru'(c_{2\alpha}) + \theta q R u'(c_{2\beta}), \\
(1-q)Ru'(c_{2\alpha}) &= \eta_\alpha, \\
qu'(c_{1\beta}) = qRu'(c_{2\beta}) &= \eta_\beta,
\end{aligned}
$$

where the last condition ensures $c_{1\beta} < c_{2\beta}$ in the solution. We let $\mathcal{A}(\theta, q) \equiv \left\{c_1^*(\theta, q), c_{2\alpha}^*(\theta, q), c_{1\beta}^*(\theta, q), c_{2\beta}^*(\theta, q)\right\}$ denote the solution to this problem. Combining the conditions, we find the following relationship:

Lemma 1 $c_{2\alpha}^*(\theta, q) > c_1^*(\theta, q)$ *always holds,* $\forall\theta, \forall q$*,*

which guarantees that a patient depositor rationally withdraws in period 2 in state α.

For the run strategy profile to constitute an equilibrium, we also need $c_1^*(\theta, q) \geq c_{2\beta}^*(\theta, q)$, both of which depend on the value of q. Specifically, they have the following relationship:

Lemma 2 $c_1^*(\theta, q) \geq c_{2\beta}^*(\theta, q)$ *if* $q \leq \bar{q}(\theta) \equiv \frac{1}{1+\frac{\theta(R-1)}{\pi(1-R\Lambda(\theta))}}$*, where* $\Lambda(\theta) = \left[\frac{1-\pi}{(\theta-\pi)R+(1-\theta)\pi R^{\frac{\gamma-1}{\gamma}}+(1-\theta)(1-\pi)}\right]^\gamma$.

The intuition behind this result is explained by the bank's incentive. When q is higher, the bank becomes more conservative against the possibility of a crisis. Then, the bank reduces c_1 to leave more resources for paying $\left\{c_{1\beta}^*, c_{2\beta}^*\right\}$. The $\bar{q}$ implies that when q is substantially high, the bank sets c_1^* even lower than $c_{2\beta}^*$, which, in turn, eliminates a depositor's incentive to run on the bank.

We use $\bar{q}$ as the measure of financial fragility, and if $c_1^*(\theta, q) \geq c_{2\beta}^*(\theta, q)$ does not hold for any value of q, then define $\bar{q} = 0$.[9] The bank is said to be *more fragile* if the $\bar{q}$ is higher. Since the solution depends on θ, the value of the $\bar{q}$ also depends on θ. Our interest is in studying how a change in θ affects $\bar{q}$.

[9] Using the maximum crisis probability as the measure of financial fragility is not novel in the literature. This approach is viewed as an application of robust control methods to the sunspot equilibrium selection. See Li (2017), Izumi (2021), and Izumi and Li (2024), for example.

4 Cost and Benefit of a structural delay

We now evaluate the equilibrium outcomes over θ to study the effect of structural delays. We first examine how the equilibrium allocation changes by holding q fixed, which helps us show the cost of a structural delay. We then study how the $\bar{q}$ changes over θ and show the possibility of a positive effect of a structural delay on fragility. Lastly, we discuss how welfare, evaluated at $\bar{q}(\theta)$, evolves over θ.

4.1 Distributional distortion

The bank's allocation determines welfare, and the allocation depends on (θ, q). Taking q as given, we study how the expected utility of depositors evaluated at $\mathcal{A}(\theta, q)$ changes over θ. When θ rises, the bank has to continue to pay c_1^* for more depositors in times of a crisis, although the state is revealed by π withdrawals. That is, until θ withdrawals, the bank cannot change its payment on the realized state. It is more efficient to adjust the amount of payments based on the realized state, and hence, an increase in θ implies that the bank has to make inefficient payments to serve more depositors. Additionally, the bank has to serve more depositors in period 1 by liquidating its assets. If there was no need to liquidate, each asset would have yielded a return of R in period 2, but liquidation results in the loss of that opportunity. Through these two channels, an increase in θ distorts the allocation.

Proposition 1 *Holding q fixed, the expected utility of depositors is monotonically decreasing in θ.*

When a faster bank run is interpreted as a structural delay, this result implies that the cost of a faster bank run is the distributional distortion through a delay in responding to a revealed state.

4.2 Fragility effects

We now turn our attention to the $\bar{q}(\theta)$. To understand the effect of a change in θ on $\bar{q}$, we first need to consider the bank's incentive. The starting point is the fact that the bank has to pay c_1^* until θ withdrawals. When θ rises, more depositors can receive $c_1^* > c_{1\beta}^*$, which is financed at the cost of the remaining depositors. The bank will have fewer resources left to pay $c_{2\beta}^*$, which pushes fragility higher. However, anticipating such a restriction, the bank chooses to decrease c_1^* to leave more resources in times of crisis. This precautionary behavior decreases fragility by making c_1^* relative to $c_{2\beta}^*$ smaller. The net effect may or may not decrease fragility.

To understand the net effect, it is useful to study the role of π in determining the $\bar{q}$. Since π represents the measure of impatient depositors, it determines how many impatient depositors are left after a suspension. As π rises, the policymaker has to liquidate more resources to make these additional repayments after the suspension, which makes fragility worse. Let $\bar{\pi}_1 \equiv \frac{R^{\frac{1}{\gamma}}-1}{R^{\frac{\gamma-1}{\gamma}}-1}$, and then we have the following result:

Lemma 3 *If $\pi > \bar{\pi}_1$, we have $\bar{q} = \frac{1}{1+\frac{\theta(R-1)}{\pi(1-R\Lambda(\theta))}}$ for $\theta \in [\pi, 1]$. If $\pi \leq \bar{\pi}_1$, we have $\bar{q} = \left\{ \begin{array}{c} \frac{1}{1+\frac{\theta(R-1)}{\pi(1-R\Lambda(\theta))}} \\ 0 \end{array} \right\}$ if $\theta \in \left\{ \begin{array}{c} (\bar{\theta}, 1] \\ [\pi, \bar{\theta}] \end{array} \right\}$, where $\bar{\theta} \equiv \frac{R\left[\pi+(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right]-\left[\pi R^{\frac{\gamma-1}{\gamma}}+(1-\pi)\right]}{R-\left[\pi R^{\frac{\gamma-1}{\gamma}}+(1-\pi)\right]} < 1$.*

Note that when $\pi > \bar{\pi}_1$, we always have $\pi > \bar{\theta}$ as shown in the proof. This lemma implies that, when $\pi \leq \bar{\pi}_1$, the bank is not fragile for any $\theta \leq \bar{\theta}$.

Using this lemma, we will report our result by dividing the parameter space into two cases: (i) $\pi \leq \bar{\pi}_1$ and (ii) $\pi > \bar{\pi}_1$. In both cases, a higher θ may decrease fragility when π is smaller. The mechanism behind this result is the same: A higher π amplifies the distributional distortion associated with an increase in θ, which reduces the depositor's payoff of withdrawing in period 2. When π is small, an increase in θ has a smaller marginal effect of distributional distortion while making the bank conservative enough. In such a case, the net effect decreases fragility. When π is large, the marginal effect of distortion is more significant, and the $\bar{q}$ is more likely to increase in θ, as shown in the following proposition:

Proposition 2 *An increase in θ may have a non-monotonic effect on $\bar{q}$:*

- *When $\pi \leq \bar{\pi}_1$, the $\bar{q}$ is $\left\{ \begin{array}{c} \text{increasing in } \theta \\ \text{increasing (decreasing) in } \theta \leq (>)\hat{\theta} \end{array} \right\}$ if $\pi \left\{ \begin{array}{c} \geq \\ < \end{array} \right\} \bar{\pi}_2$.*

- *When* $\pi > \bar{\pi}_1$*, the* $\bar{q}$ *is* $\left\{\begin{array}{c} \textit{increasing in } \theta \\ \textit{increasing (decreasing) in } \theta \leq (>)\hat{\theta} \\ \textit{decreasing in } \theta \end{array}\right\}$ *if* $\left\{\begin{array}{c} \pi \geq \bar{\pi}_2 \\ \pi < \bar{\pi}_2 \textit{ and } f(\pi) > 0 \\ f(\pi) \leq 0 \end{array}\right\}$,

where the thresholds are defined below

$$\bar{\pi}_2 = \frac{\frac{R^\gamma - R}{\gamma} - R^{\frac{1}{\gamma}} + 1}{\frac{R^\gamma - R}{\gamma} - R^{\frac{\gamma-1}{\gamma}} + 1},$$

$$\hat{\theta} = \left\{\theta \in [\pi, 1] \middle| R\Lambda(\theta) \cdot \left(1 + \gamma\theta \frac{R - \left[\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi)\right]}{(\theta - \pi)R + (1-\theta)\pi R^{\frac{\gamma-1}{\gamma}} + (1-\theta)(1-\pi)}\right) = 1\right\}, \text{ and}$$

$$f(\pi) = R\left[\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi)\right]^{-\gamma} \left\{1 + \gamma\pi \frac{R - \left[\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi)\right]}{(1-\pi)\left[\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi)\right]}\right\} - 1.$$

A numerical example is presented in Figure 1 and suggests that as θ increases, banks adopt more cautious behaviors and reduce short-term payments, which can make the system less fragile. Therefore, the limitation on a policy response caused by a faster bank run may not necessarily be bad for fragility.

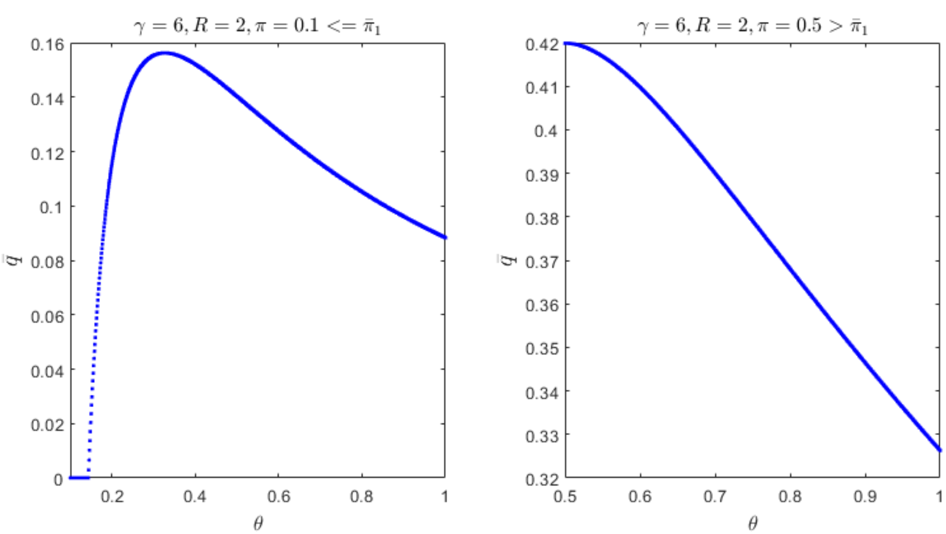


Figure 1: A delayed intervention may decrease fragility

4.3 The net outcome

We now turn our attention to the net effect: An increase in θ distorts the allocation while it may decrease fragility. In this section, we study what degree of θ achieves higher welfare, measured by the utility of depositors, by balancing these potential competing effects. To consider $\bar{q}(\theta)$ in calculating welfare, we evaluate (2) at $q = \bar{q}(\theta)$ and $\mathcal{A}(\theta, \bar{q})$, which can be expressed as

$$\begin{aligned} V(\theta, \bar{q}(\theta)) \;=\;& (1-\bar{q}(\theta))\left\{\pi u\left(c_1^*(\theta, \bar{q}(\theta))\right) + (1-\pi) u\left(c_{2\alpha}^*(\theta, \bar{q}(\theta))\right)\right\} + \\ & \bar{q}(\theta)\left\{\theta u(c_1^*(\theta, \bar{q}(\theta))) + (1-\theta)\left[\pi u(c_{1\beta}^*(\theta, \bar{q}(\theta))) + (1-\pi) u(c_{2\beta}^*(\theta, \bar{q}(\theta)))\right]\right\}. \end{aligned}$$

In this specification, an increase in θ affects not only the allocation but also the $\bar{q}(\theta)$. When $\bar{q}'(\theta) > 0, \forall\theta$, an increase in θ distorts the allocation and increases fragility:

Corollary 1 *When a delayed intervention increases fragility, a faster bank run, causing the delay, worsens welfare.*

The most interesting case thus arises when $\bar{q}'(\theta) < 0$, which may suggest that a faster bank run, or a structural delay, is desirable.

We examine the empirical plausibility of our results by showing numerical solutions. In solving the model numerically, we motivate particular values of parameters based on the literature. Specifically, we use $(\pi, R) = (0.1, 1.04)$ and will consider multiple possibilities for the value of γ, showing how the $\bar{q}$ and welfare change over θ. Our specification of the value of R is motivated by Eisenbach and Phelan (2021) and Gertler and Kiyotaki (2015), which interpret R as the return of illiquid assets. The value of π follows Eisenbach and Phelan (2021) and is also motivated by Artavanis, Paravisini, Robles Garcia, Seru, and Tsoutsoura (2022). Figure 2 illustrates how the $\bar{q}$ and welfare, evaluated at $\bar{q}$, evolve over θ. Our numerical solution shows that a delayed intervention increases fragility in this parameter set. Combining this numerical solution with Corollary 1, we conclude that, although the mechanism in which a delayed intervention may decrease fragility exists, it is not a plausible channel in the current economy.

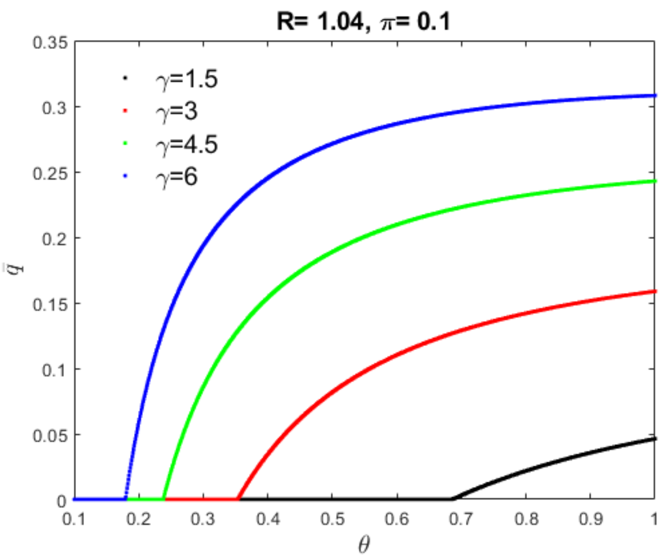


Figure 2: A delayed intervention increases fragility

5 Costly Enhancement

Choosing θ may be feasible but costly. Our motivation for $\theta \geq \pi$ is that, although policymakers must be able to infer whether a run is underway or not at π withdrawals, they may not be able to respond immediately. Such a delayed response can be caused by recognition, decision, or implementation lags. In particular, the banking turmoil in 2023 raised concerns about the faster speed of withdrawals due to online banking and social networks. Policymakers may be able to reduce response time by making additional efforts such as monitoring transactions and cash-flows better and reforming the structure of the decision-making process. These improvements, however, could be costly. In this section, we introduce the cost of changing θ, which would add an additional cost to choosing a lower θ.

In choosing θ, we assume that policymakers have to pay $\psi(\theta) = \delta(1 - \theta)$, where $\delta > 0$.[10] This function implies that the policymaker does not have to pay any costs if $\theta = 1$ but has to pay more costs in period 0 to allow an earlier intervention in period 1. We assume that this cost is financed

[10] We assume this particular function for the purpose of exposition. Our results hold as long as $\psi'(\theta) < 0$ and $\psi(1) = 0$, and we consider this general form in the proof for Proposition 3.

by the resources held by depositors in period 0, and depositors transfer $1-\psi(\theta)$ to banks in period 0.[11] Then, the bank solves (2) subject to the following modified budget constraints.

$$\pi c_1 + (1-\pi)\frac{c_{2\alpha}}{R} = 1-\psi(\theta), \tag{5}$$

$$\theta c_1 + \pi(1-\theta)c_{1\beta} + (1-\pi)(1-\theta)\frac{c_{2\beta}}{R} = 1-\psi(\theta). \tag{6}$$

The solution is summarized by $\mathcal{A}^p(\theta,q) \equiv \left\{c_1^p(\theta,q), c_{2\alpha}^p(\theta,q), c_{1\beta}^p(\theta,q), c_{2\beta}^p(\theta,q)\right\}$.[12] Evaluating (2) at $q=\bar{q}(\theta)$ and $\mathcal{A}^p(\theta,\bar{q}(\theta))$, the policymaker then chooses θ to solve the following problem.

$$\max_{\theta\in[\pi,1]} \quad \Big\{ (1-\bar{q}(\theta))\left\{\pi u\left(c_1^p(\theta,\bar{q}(\theta))\right) + (1-\pi)u\left(c_{2\alpha}^p(\theta,\bar{q}(\theta))\right)\right\} +$$
$$\bar{q}(\theta)\left\{\theta u(c_1^p(\theta,\bar{q}(\theta))) + (1-\theta)\left[\pi u(c_{1\beta}^p(\theta,\bar{q}(\theta))) + (1-\pi)u(c_{2\beta}^p(\theta,\bar{q}(\theta)))\right]\right\}\Big\}.$$

While we provide the first-order condition in the proof of Proposition 3, the solution to this problem is determined by the distributional distortion, the fragility effect, and the adjustment cost. We emphasize that when the size of the deposit changes, all the consumption levels change but only proportionally because the utility function is homothetic. As a result, the $\bar{q}$, measured by the ratio $c_1^*/c_{2\beta}^*$, remains unchanged. Thus, the optimal degree of θ will be higher as δ is higher, suggesting a delay in interventions to be optimal in such a case.

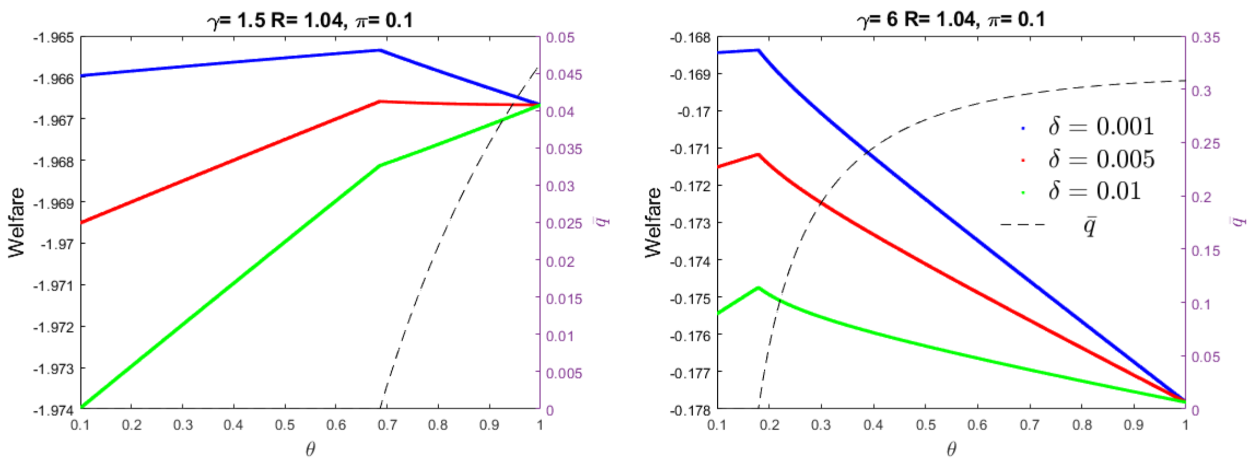
Proposition 3 *A further structural delay is desirable if the adjustment cost (δ) is higher.*

We examine the numerical solution of this problem with the adjustment cost. We assume that $\psi(\theta)=\delta(1-\theta)$, where $\delta>0$, and use the same parametric specification as in Section 4.3. Figure 3 illustrates the evolution of welfare (drawn as solid lines) and the $\bar{q}$ (drawn as a dashed line) over θ. When $\theta=1$, the value of δ does not create any differences because the policymaker does not pay any costs anyway. When $\theta<1$, a higher value of δ reduces welfare proportionally for each value

[11] This specification can be interpreted as a tax on depositors. We can alternatively assume that $\psi(\theta)$ is a welfare loss while keeping the depositor's resources unaffected, which does not change our result.

[12] Note that, because of the homothetic utility function, each consumption variable becomes proportionally smaller than $\mathcal{A}^p(\theta,q)$.

of θ. Each of these panels consistently shows that a structural delay is optimal, and the optimal degree of the structural delay depends on the value of δ. In this example, the optimal degree of θ is $\theta^* = 1$ or the point where the $\bar{q}$ starts to increase, depending on the value of δ. When $\bar{q} = 0$, welfare increases over θ just because the policymaker pays a smaller cost. When $\bar{q} > 0$ and rises, the fragility effect pushes welfare down.



(a) $\gamma = 1.5$ (b) $\gamma = 6$

Figure 3: The optimal structural delay

6 Conclusion

This paper has examined the effect of a structural delay of policy response on a bank run. The banking turmoil in 2023 shed light on the fast speed of withdrawals due to online banking and social networks, which may indicate that technologies have transformed the environment for bank runs and policy responses. This paper interpreted this as a restriction on the timing of policy responses, in which policy responses are necessarily delayed even if a policymaker desires to respond quicker ex-post. In other words, the earliest timing of intervention is constrained. We have examined how such a restriction affects equilibrium allocation and fragility, studying whether a structural delay is really bad or not.

We have examined a restriction on policy responses as in Ennis and Keister (2010), which studies a suspension policy (e.g., a deposit freeze). In their model, a policymaker can choose to suspend payments immediately after a run is inferred. In our model, there is a restriction on the timing of responses and a policymaker cannot choose to respond immediately even when a run is inferred. Such a constraint has an obvious negative effect – the size of the bank run increases, and the bank has to keep making repayments by liquidating assets.

Our main result is that such a delay can actually decrease fragility because the bank becomes more precautious. We have shown that, anticipating such a delay, banks change their repayment schedules precautionarily, leaving more resources for a crisis time. In such a case, a depositor has a weaker incentive to run on the bank. This result suggests that a delayed response may not necessarily be bad for fragility.

Finally, we studied what degree of restriction achieves higher welfare. Using empirically plausible parameter sets, we have found that our main result is not a concern in the current economy. In these parameter sets, a structural delay worsens the distributional distortions and financial fragility. We then extend our model to allow a policymaker to choose the degree of restriction ex-ante. When reducing the restriction is costly, a structural delay may be desirable.

References

Acharya, S., & Dreyfus, J.-F. (1989). Optimal bank reorganization policies and the pricing of federal deposit insurance. *The Journal of Finance*, *44*(5), 1313–1333.

Allen, L., & Saunders, A. (1993). Forbearance and valuation of deposit insurance as a callable put. *Journal of Banking and Finance*, *17*(4), 629–643.

Artavanis, N., Paravisini, D., Robles Garcia, C., Seru, A., & Tsoutsoura, M. (2022). *One size doesn't fit all: Heterogeneous depositor compensation during periods of uncertainty* (Working Paper No. 30369). National Bureau of Economic Research.

Bowman, M. W. (2024, April). *Speech by governor bowman on community banking in the age of innovation.* Retrieved from https://www.federalreserve.gov/newsevents/speech/

bowman20240403a.htm (Speech delivered on April 3, 2024)

Bruche, M., & Llobet, G. (2014). Preventing zombie lending. *The Review of Financial Studies*, *27*(3), 923–956.

Bryant, J. (1980). A model of reserves, bank runs, and deposit insurance. *Journal of Banking & Finance*, *4*(4), 335 - 344.

Diamond, D. W., & Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of political economy*, *91*(3), 401–419.

Eisenbach, T. M., & Phelan, G. (2021). Cournot fire sales in real and financial markets. *American Economic Journal: Macroeconomics*, *forthcoming*.

Ennis, H. M., & Keister, T. (2009). Bank runs and institutions: The perils of intervention. *American Economic Review*, *99*(4), 1588–1607.

Ennis, H. M., & Keister, T. (2010). Banking panics and policy responses. *Journal of Monetary Economics*, *57*(4), 404–419.

Faria-e-Castro, M., Martinez, J., & Philippon, T. (2017). Runs versus lemons: Information disclosure and fiscal capacity. *The Review of Economic Studies*, *84*(4 (301)), 1683–1707.

Gao, J., & Reed, R. R. (n.d.). The size distribution of the banking sector and financial fragility. *Journal of Money, Credit and Banking*.

Gertler, M., & Kiyotaki, N. (2015, July). Banking, liquidity, and bank runs in an infinite horizon economy. *American Economic Review*, *105*(7), 2011-43.

Green, E. J., & Lin, P. (2003). Implementing efficient allocations in a model of financial intermediation. *Journal of Economic Theory*, *109*(1), 1-23.

Izumi, R. (2021). Opacity: Insurance and fragility. *Review of Economic Dynamics*, *40*, 146-169.

Izumi, R., & Li, Y. (2024). Financial stability with fire sale externalities. *mimeo*.

Keister, T. (2016). Bailouts and financial fragility. *Review of Economic Studies*, *83*, 89–104.

Keister, T., & Mitkov, Y. (2024). Allocating losses: Bail-ins, bailouts and bank regulation. *Journal of Economic Theory*, *210*.

Keister, T., & Narasiman, V. (2016). Expectations vs. fundamentals-driven bank runs: When should bailouts be permitted? *Review of Economic Dynamics*, *21*, 89-104.

Koenig, P., & Mayer, P. (2022). Optimal timing of policy interventions in troubled banks. *Deutsche Bundesbank Discussion Paper*(10/2022).

Li, Y. (2017). Interest rates and financial fragility. *Journal of Economic Dynamics and Control*.

Mailath, G. J., & Mester, L. J. (1994). A positive analysis of bank closure. *Journal of Financial Intermediation*, *3*(3), 272–299.

Morrison, A. D., & White, L. (2013). Reputational contagion and optimal regulatory forbearance. *Journal of Financial Economics*, *110*(3), 642–658.

Peck, J., & Shell, K. (2003). Equilibrium bank runs. *Journal of Political Economy*, *111*(1), 103–123.

Philippon, T., & Skreta, V. (2012). Optimal interventions in markets with adverse selection. *American Economic Review*, *102*(1), 1–28.

Rose, J. D. (2023, May). Understanding the speed and size of bank runs in historical comparison. *Economic Synopses*(12), 1-5.

Schilling, L. (in press). Optimal forbearance of bank resolution. *Journal of Finance*.

Segura, A., & Suarez, J. (2019). Optimally solving banks' legacy problems. *Bank of Italy Temi di Discussione (Working Paper)*(1227).

Wallace, N. (1988). Another attempt to explain an llliquid banking system: With sequential service taken seriously. *Federal Reserve Bank of Minneapolis Quarterly Review*, *12*, 3-16.

Wallace, N. (1990). A banking model in which partial suspension is best. *Federal Reserve Bank of Minneapolis Quarterly Review*, *14*, 11-23.

Walther, A. (2016). Jointly optimal regulation of bank capital and liquidity. *Journal of Money, Credit and Banking*, *48*(2-3), 415-448.

A Proofs of Propositions

Lemma 1 $c^*_{2\alpha}(\theta, q) > c^*_1(\theta, q)$ *always holds,* $\forall\theta, \forall q$,

Proof. Given the first-order conditions derived in Section 3, we have

$$[(1-q)\pi + q\theta]\, u'(c_1) = (1-q)\pi R u'(c_{2\alpha}) + q\theta R u'(c_{2\beta}),$$

which yields

$$(1-q)\pi R\left(\frac{\frac{1}{c_1}-\pi}{(1-\pi)\frac{1}{R}}\right)^{-\gamma} + q\theta R\left(\frac{\frac{1}{c_1}-\theta}{(1-\theta)\frac{1}{R}[\pi R^{1-1/\gamma}+(1-\pi)]}\right)^{-\gamma} = [(1-q)\pi + q\theta].$$

Define

$$f(c_1) = (1-q)\pi R\left(\frac{\frac{1}{c_1}-\pi}{(1-\pi)\frac{1}{R}}\right)^{-\gamma} + q\theta R\left(\frac{\frac{1}{c_1}-\theta}{(1-\theta)\frac{1}{R}[\pi R^{1-1/\gamma}+(1-\pi)]}\right)^{-\gamma} - [(1-q)\pi + q\theta]$$

It is easy to see that the function $f(c_1)$ is strictly increasing in $c_1 \in (0, 1/\theta)$, and that $f(c_1^*(\theta, q)) = 0$.

To show that $c_{2\alpha}^*(\theta, q) > c_1^*(\theta, q)$ always holds, given the resource constraint (3), we need $c_1^*(\theta, q) < 1/(\pi + (1-\pi)/R)$ is satisfied. Combined with the shape of the function $f(c_1)$, the above necessary condition can be rewritten as:

$$f(c_1 = 1/(\pi+(1-\pi)/R)) = (1-q)\pi R + q\theta R\left(\frac{\pi + (1-\pi)/R - \theta}{(1-\theta)\frac{1}{R}[\pi R^{1-1/\gamma}+(1-\pi)]}\right)^{-\gamma} - [(1-q)\pi + q\theta] > 0.$$

This condition has always held since the term $\frac{\pi+(1-\pi)/R-\theta}{(1-\theta)\frac{1}{R}[\pi R^{1-1/\gamma}+(1-\pi)]}$ is no more than 1, combined with the fact that $R > 1$, which implies that $(1-q)\pi R + q\theta R\left(\frac{\pi+(1-\pi)/R-\theta}{(1-\theta)\frac{1}{R}[\pi R^{1-1/\gamma}+(1-\pi)]}\right)^{-\gamma} > (1-q)\pi + q\theta R$. In other words, the condition $f(c_1 = 1/(\pi + (1-\pi)/R)) > 0$ always holds. It then gives us the desired result of $c_{2\alpha}^*(\theta, q) > c_1^*(\theta, q)$. ■

Lemma 2 *$c_1^*(\theta, q) \geq c_{2\beta}^*(\theta, q)$ if $q \leq \bar{q}(\theta) \equiv \frac{1}{1+\frac{\theta(R-1)}{\pi(1-R\Lambda(\theta))}}$, where $\Lambda(\theta) = \left[\frac{1-\pi}{(\theta-\pi)R+(1-\theta)\pi R^{\frac{\gamma-1}{\gamma}}+(1-\theta)(1-\pi)}\right]^{\gamma}$.*

Proof. As stated in Lemma 1, the optimal value of $c_1^*(\theta, q)$ is determined by $f(c_1^*(\theta, q)) = 0$. To find the condition such that $c_1^*(\theta, q) \geq c_{2\beta}^*(\theta, q)$ holds, given the resource constraint (4), we need $c_1^*(\theta, q) \geq 1/\left\{\theta + (1-\theta)/R[\pi R^{1-1/\gamma} + (1-\pi)]\right\}$ is satisfied. Combined with the shape of the function $f(c_1)$, the above sufficient condition can be rewritten as:

$$f\left(c_1 = \frac{1}{\theta + (1-\theta)\frac{1}{R}[\pi R^{1-1/\gamma} + (1-\pi)]}\right) = (1-q)\pi R\Lambda(\theta) + q\theta R - [(1-q)\pi + q\theta] \leq 0.$$

In other words, the condition $c_1^*(\theta, q) \geq c_{2\beta}^*(\theta, q)$ holds, if $q \leq \bar{q}(\theta) \equiv \frac{1}{1+\frac{\theta(R-1)}{\pi(1-R\Lambda(\theta))}}$. ■

Proposition 1 *Holding q fixed, the expected utility of depositors is monotonically decreasing in θ.*

Proof. Given q, the expected utility of depositors is given as:

$$W^*(\theta, q) = (1-q)[\pi u(c_1^*(\theta, q)) + (1-\pi)u(c_{2\alpha}^*(\theta, q))] + q\left\{\theta u(c_1^*(\theta, q)) + (1-\theta)[\pi R^{1-1/\gamma} + (1-\pi)]u(c_{2\beta}^*(\theta, q))\right\}.$$

Taking the differentiate of θ on both sides, we have

$$\begin{aligned}\frac{dw^*(\theta, q)}{d\theta} &= \left\{[(1-q)\pi + q\theta]u'(c_1^*(\theta, q)) - (1-q)\pi R u'(c_{2\alpha}^*(\theta, q)) - q\theta R u'(c_{2\beta}^*(\theta, q))\right\}\frac{dc_1^*(\theta, q)}{d\theta} \\ &+ q\left\{u(c_1^*(\theta, q)) - [\pi R^{1-1/\gamma} + (1-\pi)]u(c_{2\beta}^*(\theta, q)) + Ru'(c_{2\beta}^*(\theta, q))\frac{1-c_1^*(\theta, q)}{1-\theta)}\right\}.\end{aligned}$$

Recall that the first-order condition $(1-q)\pi R\left(\frac{\frac{1}{c_1}-\pi}{(1-\pi)\frac{1}{R}}\right)^{-\gamma} + q\theta R\left(\frac{\frac{1}{c_1}-\theta}{(1-\theta)\frac{1}{R}[\pi R^{1-1/\gamma}+(1-\pi)]}\right)^{-\gamma} = [(1-q)\pi + q\theta]$, $\frac{dw^*(\theta, q)}{d\theta}$ turns out to be:

$$\frac{dw^*(\theta, q)}{d\theta} = q\left\{u(c_1^*(\theta, q)) - [\pi R^{1-1/\gamma} + (1-\pi)]u(c_{2\beta}^*(\theta, q)) + Ru'(c_{2\beta}^*(\theta, q))\frac{1-c_1^*(\theta, q)}{1-\theta)}\right\}.$$

Next, recall that the resource constraint (4) and the preference of CRRA form, $\frac{dw^*(\theta, q)}{d\theta}$ can be rewritten as:

$$\frac{dw^*(\theta, q)}{d\theta} = qu(c_1^*(\theta, q)) \cdot A(c_1^*(\theta, q)),$$

where

$$A(c_1^*(\theta,q)) = \left\{1 + (1-\theta)^{\gamma-1} R^{1-\gamma}[\pi R^{1-1/\gamma} + (1-\pi)]^{\gamma} \cdot B\left(\frac{1}{c_1^*(\theta,q)}\right)\right\},$$

$$B\left(\frac{1}{c_1^*(\theta,q)}\right) = \left(\frac{1}{c_1^*(\theta,q)} - \theta\right)^{-\gamma} \cdot \left[(\gamma - 1 + \theta) - \frac{\gamma}{c_1^*(\theta,q)}\right].$$

It is straightforward to show that the function $B\left(\frac{1}{c_1^*(\theta,q)}\right)$ is strictly decreasing in $1/c_1^*(\theta,q)$. In other words, $B\left(\frac{1}{c_1^*(\theta,q)}\right)$ is strictly increasing in $c_1^*(\theta,q)$. Therefore, $A(c_1^*(\theta,q))$ is strictly increasing in $c_1^*(\theta,q)$.

In equilibrium, the condition $c_1^*(\theta,q) \geq c_{2\beta}^*(\theta,q)$ holds, which in turn yields

$$c_1^*(\theta,q) \geq \frac{1}{\left\{\theta + (1-\theta)\frac{1}{R}[\pi R^{1-1/\gamma} + (1-\pi)]\right\}} > 1.$$

Thus,

$$A(c_1^*(\theta,q)) > A(c_1^*(\theta,q))|_{c_1^*(\theta,q)=1} = 1 - R^{1-\gamma}[\pi R^{1-1/\gamma} + (1-\pi)]^{\gamma} > 0,$$

which implies that $\frac{dw^*(\theta,q)}{d\theta} < 0$ as desired. ■

Lemma 3 *If $\pi > \bar{\pi}_1$, we have $\bar{q} = \frac{1}{1+\frac{\theta(R-1)}{\pi(1-R\Lambda(\theta))}}$ for $\theta \in [\pi, 1]$. If $\pi \leq \bar{\pi}_1$, we have $\bar{q} = \left\{\begin{array}{c} \frac{1}{1+\frac{\theta(R-1)}{\pi(1-R\Lambda(\theta))}} \\ 0 \end{array}\right\}$ if $\theta \in \left\{\begin{array}{c} (\bar{\theta}, 1] \\ [\pi, \bar{\theta}] \end{array}\right\}$, where $\bar{\theta} \equiv \frac{R\left[\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}\right] - \left[\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi)\right]}{R - \left[\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi)\right]} < 1$.*

Proof. It is easy to see that if $\pi > \bar{\pi}_1$ then $R\Lambda(\theta) < 1$ always holds, which implies that $\bar{q} > 0$ for $\theta \in [\pi, 1]$. Now, suppose $\pi \leq \bar{\pi}_1$, we have $R\Lambda(\theta) \left\{\begin{array}{c} < \\ \geq \end{array}\right\} 1$ if $\theta \in \left\{\begin{array}{c} (\bar{\theta}, 1] \\ [\pi, \bar{\theta}] \end{array}\right\}$, gives our desired result. ■

Proposition 2 *An increase in θ may have a non-monotonic effect on $\bar{q}$:*

- *When $\pi \leq \bar{\pi}_1$, the $\bar{q}$ is $\left\{\begin{array}{c} \text{increasing in } \theta \\ \text{increasing (decreasing) in } \theta \leq (>)\hat{\theta} \end{array}\right\}$ if $\pi \left\{\begin{array}{c} \geq \\ < \end{array}\right\} \bar{\pi}_2$.*

- *When* $\pi > \bar{\pi}_1$, *the* $\bar{q}$ *is* $\left\{\begin{array}{c} \textit{increasing in } \theta \\ \textit{increasing (decreasing) in } \theta \leq (>) \hat{\theta} \\ \textit{decreasing in } \theta \end{array}\right\}$ *if* $\left\{\begin{array}{c} \pi \geq \bar{\pi}_2 \\ \pi < \bar{\pi}_2 \textit{ and } f(\pi) > 0 \\ f(\pi) \leq 0 \end{array}\right\}$,

Proof. In the first place, we assume $\pi \leq \bar{\pi}_1$, and thus Lemma 3 tells us that $\bar{q} = \left\{\begin{array}{c} \frac{1}{1+\frac{\theta(R-1)}{\pi(1-R\Lambda(\theta))}} \\ 0 \end{array}\right\}$ if $\theta \in \left\{\begin{array}{c} (\bar{\theta}, 1] \\ [\pi, \bar{\theta}] \end{array}\right\}$. After differentiating $\bar{q}$ with respect to θ, it is proportional to:

$$\frac{d\bar{q}}{d\theta} \propto R\Lambda(\theta)\left(1 + \gamma\theta \frac{R - \left[\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi)\right]}{(\theta-\pi)R + (1-\theta)\pi R^{\frac{\gamma-1}{\gamma}} + (1-\theta)(1-\pi)}\right) - 1 \equiv B(\theta).$$

Similarly, after differentiating $B(\theta)$ with respect to θ, it is proportional to:

$$\begin{aligned}
\frac{dB(\theta)}{d\theta} &\propto -\gamma\Lambda(\theta)\frac{R - \left[\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi)\right]}{(\theta-\pi)R + (1-\theta)\pi R^{\frac{\gamma-1}{\gamma}} + (1-\theta)(1-\pi)}\left(1 + \gamma\theta\frac{R - \left[\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi)\right]}{(\theta-\pi)R + (1-\theta)\pi R^{\frac{\gamma-1}{\gamma}} + (1-\theta)(1-\pi)}\right) \\
&\quad + \gamma\Lambda(\theta)\frac{R - \left[\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi)\right]}{(\theta-\pi)R + (1-\theta)\pi R^{\frac{\gamma-1}{\gamma}} + (1-\theta)(1-\pi)}\frac{\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi) - \pi R}{(\theta-\pi)R + (1-\theta)\pi R^{\frac{\gamma-1}{\gamma}} + (1-\theta)(1-\pi)} \\
&\propto \frac{\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi) - \pi R}{(\theta-\pi)R + (1-\theta)\pi R^{\frac{\gamma-1}{\gamma}} + (1-\theta)(1-\pi)} - \left(1 + \gamma\theta\frac{R - [\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi)]}{(\theta-\pi)R + (1-\theta)\pi R^{\frac{\gamma-1}{\gamma}} + (1-\theta)(1-\pi)}\right) \\
&\propto -(\gamma+1)\theta\left\{R - \left[\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi)\right]\right\} < 0.
\end{aligned}$$

Recall that $\bar{q} = 0$ as $\theta \in [\pi, \bar{\theta}]$ if $\pi \leq \bar{\pi}_1$. Thus, we need to see the shape of $\bar{q}$ in $\theta \in (\bar{\theta}, 1]$. We know that $B(\theta)$ is decreasing in θ and thus $B(\theta)$ is decreasing in $\theta(\bar{\theta}, 1]$ for sure. Next, we first calculate the values of $B(\theta)$ at $\theta = \bar{\theta}$ and 1, and then we can determine the sign of $\frac{d\bar{q}}{d\theta}$. After some algebra, we have $A|_{\theta=\bar{\theta}} = \frac{1}{R}$, and $A|_{\theta=1} = R^{-\gamma}$. These two values yield

$$B(\theta)|_{\theta=\bar{\theta}} = \left(1 + \gamma\bar{\theta}\frac{R - (\pi R^{\frac{\gamma-1}{\gamma}} + (1-\pi))}{(\bar{\theta}-\pi)R + (1-\bar{\theta})\pi R^{\frac{\gamma-1}{\gamma}} + (1-\bar{\theta})(1-\pi)}\right) - 1 > 0,$$

and

$$B(\theta)|_{\theta=1} = R^{1-\gamma}\left(1-\theta\frac{R-[\pi R^{\frac{\gamma-1}{\gamma}}+(1-\pi)]}{(1-\pi)R}\right)-1 \propto \pi\left(\frac{R^{\gamma}-R}{\gamma}-R^{\frac{\gamma-1}{\gamma}}+1\right)-\left(\frac{R^{\gamma}-R}{\gamma}-R+1\right).$$

Note that $\frac{d}{dR}\left(\frac{R^{\gamma}-R}{\gamma}-R^{\frac{\gamma-1}{\gamma}}+1\right)=\frac{1}{\gamma}(\gamma R^{\gamma-1}-1)-(1-\frac{1}{\gamma})R^{-\frac{1}{\gamma}}$, and that $\frac{d^2}{dR^2}\left(\frac{R^{\gamma}-R}{\gamma}-R^{\frac{\gamma-1}{\gamma}}+1\right)=(\gamma-1)R^{\gamma-2}+\frac{1}{\gamma}(1-\frac{1}{\gamma})R^{-\frac{1}{\gamma}-1}>0$. Thus, $\frac{d}{dR}\left(\frac{R^{\gamma}-R}{\gamma}-R^{\frac{\gamma-1}{\gamma}}+1\right)>\frac{1}{\gamma}(\gamma-1)-(1-\frac{1}{\gamma})=0$, which in turn implies that $\frac{R^{\gamma}-R}{\gamma}-R^{\frac{\gamma-1}{\gamma}}+1>\frac{1-1}{\gamma}-1+1=0$. Therefore, $B(\theta)\mid_{\theta=1}\geq 0$ always holds as long as $\pi \geq \frac{\frac{R^{\gamma}-R}{\gamma}-R+1}{\frac{R^{\gamma}-R}{\gamma}-R^{\frac{\gamma-1}{\gamma}}+1}\equiv \bar{\pi}_2$.

Recall that $\frac{dB(\theta)}{d\theta}<0$ and $B(\theta)\mid_{\theta=\bar{\theta}}>0$, combined with $B(\theta)\mid_{\theta=1}\geq 0$ if $\pi\geq\bar{\pi}_2$ holds, we have $B(\theta)\geq 0$. This fact then gives us, $\frac{d\bar{q}}{d\theta}\propto B(\theta)\geq 0$, if $\pi\geq\bar{\pi}_2$ holds.

Now, focus on the case where $\pi<\bar{\pi}_2$, which yields $B(\theta)\mid_{\theta=1}<0$, combined with $B(\theta)\mid_{\theta=\bar{\theta}}>0$, we have a unique $\hat{\theta}=\{\theta\in[\pi,1]|B(\theta)=0\}$ such that $B(\theta)\geq(<)0$ if $\theta\leq(>)\hat{\theta}$. In other words, $\bar{q}$ is increasing (decreasing) as $\theta\leq(>)\hat{\theta}$ if $\pi<\bar{\pi}_2$.

We next assume that $\pi>\bar{\pi}_1$, and then we have $\bar{q}=\frac{1}{1+\frac{\theta(R-1)}{\pi(1-R\Lambda(\theta))}}$ for $\theta\in[\pi,1]$. Recall that $\frac{dB(\theta)}{d\theta}<0$ for $\theta[\pi,1]$. As before, we first calculate $B(\theta)\mid_{\theta=\pi}$ and $B(\theta)\mid_{\theta=1}$ and we have

$$B(\theta)\mid_{\theta=\pi}=R(\pi R^{\frac{\gamma-1}{\gamma}}+(1-\pi))^{\gamma}\left(1+\gamma\pi\frac{R-(\pi R^{\frac{\gamma-1}{\gamma}}+(1-\pi))}{(1-\pi)(\pi R^{\frac{\gamma-1}{\gamma}}+(1-\pi))}\right)-1\equiv f(\pi),$$

and

$$B\mid_{\theta=1}=R^{1-\gamma}\left(1+\gamma\frac{R-(\pi R^{\frac{\gamma-1}{\gamma}}+(1-\pi))}{(1-\pi)R}\right)-1.$$

After some algebra, it is easy to see that $B(\theta)\left\{\begin{array}{c}\geq 0\ in\ \theta\\ \geq(<)0\ as\ \theta\leq(>0)\hat{\theta}\\ \leq 0\ in\ \theta\end{array}\right\}$ if $\left\{\begin{array}{c}B\mid_{\theta=1}\geq 0\\ B\mid_{\theta=\pi}=f(\pi)>0\ and\ B\mid_{\theta=1}<0\\ B\mid_{\theta=\pi}=f(\pi)\leq 0\end{array}\right\}$

In other words, we have

$$\bar{q} \text{ is } \left\{ \begin{array}{c} \textit{increasing in } \theta \\ \textit{increasing(decreasing) as } \theta \geq (>0)\hat{\theta} \\ \textit{decreasing in } \theta \end{array} \right\} \text{ if } \left\{ \begin{array}{c} \pi \geq \bar{\pi}_2 \\ \pi < \bar{\pi}_2 \textit{ and } f(\pi) > 0 \\ f(\pi) \leq 0 \end{array} \right\}. \blacksquare$$

Corollary 1 *When a delayed intervention increases fragility, a faster bank run, causing the delay, worsens welfare.*

Proof. Differentiate $V(\theta, \bar{q}(\theta))$ with respect to θ, we have

$$\frac{dV(\theta, \bar{q}(\theta))}{d\theta} = \left[\bar{q}(\theta) \cdot D + \frac{d\bar{q}(\theta)}{d\theta} \cdot E(\theta) \right] u(c_1^*(\theta, \bar{q}(\theta))),$$

where

$$c_1^*(\theta, \bar{q}(\theta)) = \frac{1}{\theta + (1-\theta)\frac{1}{R}[\pi R^{1-1/\gamma} + (1-\pi)]}$$

$$D = 1 + R\left(\gamma \left\{ 1 - \frac{1}{R}[\pi R^{1-1/\gamma} + (1-\pi)] \right\} - 1\right)$$

$$E(\theta) = \left\{ (\theta - \pi) + (1-\theta)[\pi R^{1-1/\gamma} + (1-\pi)] \right\} - (1-\pi) \left\{ \frac{(\theta-\pi) + (1-\theta)\frac{1}{R}[\pi R^{1-1/\gamma} + (1-\pi)]}{(1-\pi)/R} \right\}^{1-\gamma}.$$

It is easy to see that the term D is always positive, as shown in Proposition 1, and that $E(\theta)$ is always positive as well. This fact implies that $\frac{dV(\theta, \bar{q}(\theta)}{d\theta}$ is always negative as long as $\frac{d\bar{q}(\theta)}{d\theta}$ is positive.

■

Proposition 3 *A further structural delay is desirable if the adjustment cost (δ) is higher.*

Proof. Similar to Corollary 1, differentiating $V^p(\theta, \bar{q}(\theta))$ with respect to θ yields:

$$\frac{dV^p(\theta, \bar{q}(\theta))}{d\theta} = \left[\bar{q}(\theta) \cdot D + \frac{d\bar{q}(\theta)}{d\theta} \cdot E(\theta) \right] u(c_1^p(\theta, \bar{q}(\theta))) - \psi'(\theta) \cdot F(\theta) u'(c_1^p(\theta, \bar{q}(\theta))),$$

where

$$
\begin{aligned}
c_1^p(\theta,\bar{q}(\theta)) &= \frac{1-\psi(\theta)}{\theta+(1-\theta)\frac{1}{R}[\pi R^{1-1/\gamma}+(1-\pi)]} \\
D &= 1+R\left(\gamma\left\{1-\frac{1}{R}[\pi R^{1-1/\gamma}+(1-\pi)]\right\}-1\right) \\
E(\theta) &= \left\{(\theta-\pi)+(1-\theta)[\pi R^{1-1/\gamma}+(1-\pi)]\right\}-(1-\pi)\left\{\frac{(\theta-\pi)+(1-\theta)\frac{1}{R}[\pi R^{1-1/\gamma}+(1-\pi)]}{(1-\pi)/R}\right\}^{1-\gamma} \\
F(\theta) &= \left\{\frac{(\theta-\pi)+(1-\theta)\frac{1}{R}[\pi R^{1-1/\gamma}+(1-\pi)]}{(1-\pi)/R}\right\}^{-\gamma}.
\end{aligned}
$$

As we emphasized in the main text, when the size of deposits changes, all the consumption levels change but only proportionally because the utility function is homothetic. As a result, the $\bar{q}(\theta)$, measured by the ratio, remains unchanged. The same fact also applies to the value of $\frac{d\bar{q}(\theta)}{d\theta}$.

Compared to the value of $\frac{dV(\theta,\bar{q}(\theta))}{d\theta}$, it is easy to see that the extra term $-\psi'(\theta)\cdot F(\theta)u'(c_1^p(\theta,\bar{q}(\theta)))$ is always positive and that a higher $-\psi'(\theta)$ will lead to a larger $\frac{dV^p(\theta,\bar{q}(\theta))}{d\theta}$. This implies that the optimal degree of θ^p will be higher than θ^*.

In the main text, we consider a specific cost function in the form of $\psi(\theta)=\delta(1-\theta)$. Thus, the term $-\psi'(\theta)$ turns out to be δ. This result demonstrates that a larger increase in δ may lead to a higher θ^p. ■