Wesleyan Economic Working Papers

http://repec.wesleyan.edu/
N°: 2025-004

Bank Runs and Policy Interventions under Uncertainty

Jennie Ebihara & Ryuichiro Izumi

May 2025

WESLEYAN UNIVERSITY

Department of Economics
Public Affairs Center
238 Church Street
Middletown, CT 06459-007

Tel: (860) 685-2340
Fax: (860) 685-2301
http://www.wesleyan.edu/econ

Bank Runs and Policy Interventions under Uncertainty*

Jennie Ebihara
Wesleyan University
jebihara@wesleyan.edu

Ryuichiro Izumi
Wesleyan University
rizumi@wesleyan.edu

May 5, 2025

Abstract

We study how the speed of withdrawals affects bank fragility by examining two dimensions: unpredictability in outflows and frictions in the timing of policy intervention. We extend Ennis and Keister (2009) by introducing uncertainty into the policymaker's ex-post suspension problem. When withdrawals are more unpredictable, the policymaker intervenes earlier, making the bank less fragile. In contrast, the frequency of intervention opportunities may have a non-monotonic effect: a modest decrease delays suspension and increases fragility, but when opportunities become sufficiently infrequent, the authority suspends earlier to avoid costly delays.

Keywords: Fast bank runs; Suspensions; Ex-post optimal intervention

JEL classification: G21, G28, E58

*We thank Luis Araujo, John Bonin, Richard Grossman, Masami Imai, Todd Keister, Gilbert Skillman, Peter Rutland, and seminar participants at Wesleyan University and the PIER 2024 conference at Williams College. All errors are ours.

1 Introduction

The speed at which withdrawals occur has become a central concern in understanding financial fragility. While the mechanism of bank runs have long been studied, recent episodes highlight how digital technologies and social networks can amplify both the pace and unpredictability of withdrawals.[1] For instance, depositors at Silicon Valley Bank attempted to withdraw nearly 87 percent of total deposits within 48 hours, sharply compressing the timeline before available funds were exhausted. Such rapid outflows leave little room for authorities to intervene effectively. This paper asks: How does the speed of withdrawals affect bank fragility through its impact on the effectiveness of policy intervention? We approach this question by distinguishing two ways in which speed matters. We first interpret the pace of withdrawals as that policymakers can act only at limited moments, due to delays in receiving information, coordinating decisions, or initiating a suspension. Another interpretation is that panic spreads so suddenly, driven by viral information or social contagion, that all remaining depositors attempt to exit at once, leaving no opportunity to respond.[2] Though the mechanisms differ, both forms of fast withdrawals constrain the timing of intervention. We study how increased withdrawal speed alters the effectiveness of suspension policies and, in turn, affects the fragility of the banking system.

To study how withdrawal speed influences bank fragility, we extend the model developed by Ennis and Keister (2009), where a policymaker can suspend payments but cannot pre-commit to future actions. In that environment, the policymaker decides when to suspend payments ex-post efficiently. We introduce two key modifications to this framework. First, we impose frictions on the timing of intervention to capture practical constraints, such as delays in monitoring, coordination, or supervisory review, that become more acute when withdrawals are fast. Second, we allow withdrawals to suddenly accelerate, capturing the risk that the loss of confidence spreads quickly and all remaining depositors may attempt to exit at once. This extension introduces uncertainty into the

[1] Rose (2023) highlights that recent bank runs, such as those in 2022 and 2023, were extraordinarily fast and large compared to historical standards. Technological advancements, social media, and a concentrated base of uninsured depositors contributed to this unprecedented speed.

[2] Cookson, Fox, Gil-Bazo, Imbet, and Schiller (2023) find that, during the 2023 banking turmoil, Twitter helped spread information and amplified withdrawal pressure.

policymaker's decision problem, and we use this model to examine how faster or more unpredictable outflows affect the timing and effectiveness of intervention and, ultimately, the fragility of the banking system.

A benevolent policymaker observes the pace of withdrawals and decides sequentially whether to suspend payments. These decisions are made under uncertainty: at each opportunity to intervene, the authority does not know whether future opportunities will arise. Suspending early can stop further liquidation, but doing so may leave others, who have immediate liquidity needs, unpaid. Delaying a suspension, on the other hand, allows more depositors to be served, but more assets will be liquidated, and the chance to suspend may be lost. This trade-off shapes the timing of intervention and, in turn, depositor incentives. If depositors expect timely suspension, those without urgent liquidity needs may choose to wait, avoiding a run. If not, they have an incentive to withdraw immediately. Our analysis focuses on how this decision-making depends on the speed and unpredictability of withdrawals.

We begin our analysis with a deterministic case in which the policymaker has an opportunity to suspend payments after each fixed amount of withdrawals. This environment still differs from Ennis and Keister (2009) because the decision nodes of the policy makers are still limited. Without such a friction, the authority would like to stop withdrawals at a precise point that balances help for depositors with liquidity needs against costly liquidation, yet there is no guarantee that the policymaker gets a chance to suspend payments at this exact point. Thus, the policymaker must decide whether to halt one block earlier or one block later than this ideal point, where each block represents the amount of withdrawals that occurs between two possible intervention opportunities. We compare the value of suspending with that of continuing (no-suspension) in order to identify the suspension point with this friction. Whether a bank run occurs depends on whether this suspension happens early enough to prevent patient depositors from anticipating losses. A run occurs if the suspension is expected to come too late.

We then turn to the stochastic case in which withdrawals may suddenly accelerate, eliminating future opportunities to suspend. This setup introduces uncertainty into the policymaker's decision problem and creates an additional trade-off: waiting may allow more impatient depositors to be served, but it risks forfeiting the chance to suspend payments. We show how this risk influences the authority's incentives and can lead to earlier intervention. In particular, when the chance of a sudden, full-scale run is high, the authority may become more willing to suspend earlier because the value of continuation becomes smaller, even if doing so means cutting off depositors with liquidity needs. As a result, greater volatility may reduce fragility.

An interesting result is that the frequency of suspension opportunities can have a non-monotonic effect on fragility. When opportunities are frequent, stopping later sacrifices few resources while allowing another set of depositors with liquidity needs to be paid, so the authority delays, giving rise to a bank run. In contrast, when opportunities are already infrequent, a further decrease in frequency makes the bank less fragile. In this case, serving an extra block would cause substantial liquidation, making another delay too costly. The policymaker thus suspends payments earlier, which in turn eliminates a bank run. A decrease in frequency caused by faster withdrawals can therefore increase fragility when opportunities are frequent, but reduce fragility when they are infrequent.

Our findings highlight that the relationship between withdrawal speed and fragility is nontrivial. Faster outflows can lead to worse outcomes by narrowing the window for suspension, but they can also induce quicker policy responses that prevent runs. The net effect depends on the timing constraints faced by policymakers. These insights help interpret recent episodes and suggest that improving the frequency of intervention opportunities does not always decrease fragility. If the authority delays action because it expects more opportunities, the outcome can be worse.

This paper contributes to a growing literature on the timing of policy intervention during fast bank runs. Our work builds directly on Ennis and Keister (2009), who study ex post suspension decisions in a deterministic environment. We extend their framework by introducing uncertainty in withdrawal dynamics and constraints on how frequently the policymaker can act. Our analysis is also related to Izumi and LI (2024), who examine faster bank runs in a version of the Diamond and Dybvig (1983) model without commitment. In their model, the timing of intervention is exogenous, and the policymaker is assumed to act as early as possible. We instead endogenize the timing decision and study how the authority chooses when to suspend under uncertainty. We also complement Shy (2025), who studies how faster withdrawals affect optimal bailouts and liquidity requirements. Finally, our work relates to Philipp J. König and Pothier (2024), who analyzes policy dynamics under uncertainty about bank solvency. We contribute to this discussion by showing how withdrawal speed and unpredictability shape intervention incentives, influence depositor behavior, and affect fragility.

The remainder of the paper is organized as follows. Section 2 introduces the model based on Ennis and Keister (2009) with two key modifications: intermittent intervention opportunities and stochastic withdrawal dynamics. Section 3 analyzes the benchmark case in which withdrawals follow a predictable, gradual path. We characterize the authority's suspension decision and identify conditions under which a run equilibrium arises. Section 4 introduces uncertainty into outflows, capturing the possibility of sudden full-scale withdrawals. We show how this risk reshapes the policymaker's incentives and affects fragility, followed by discussions on the broader implications of our results.

2 The Environment

Our model is based on Ennis and Keister (2009), where the policymaker chooses the point of suspension ex-post optimally. We augment the model to add uncertainty into the policymaker's ex-post decision.

2.1 The Basic Model

Time and Agents: There are three time periods, denoted by $t = 0, 1, 2$ and a mass 1 of continuum of depositors, indexed by $i \in [0, 1]$.

Depositor Preferences: Each depositor is provided a unit of endowment good in period 0 and nothing in period 1 or 2. The preference of a depositor is given by

$$u(c_1, c_2; \omega_i) = u(c_1 + \omega_i c_2),$$

where u is strictly increasing, strictly concave, and satisfies $u(0) = 0$ and $u'(0) = +\infty$. We let c_t^i represent consumption in period t, and the parameter $\omega_i \in \{0, 1\}$ is a binominal random variable. The value of ω_i realizes at the beginning of period 1 and is privately observed by each depositor. If $\omega_i = 1$, depositor i values consumption in periods 1 and 2, who we call patient. If $\omega_i = 0$, depositor i values consumption only in period 1, who we call impatient. Each depositor is chosen to be impatient with a probability $\pi \in (0, 1)$, and the fraction of impatient depositors is π while the fraction of patient depositors is $1 - \pi$. The value of π is known and constant.

Technology: There are two single, constant-returns-to-scale technologies for transforming endowment good into consumption in the later periods. One unit of the good placed into *investment* in period 0 yields $r < 1$ unit in period 1 and $R > 1$ units in period 2. A unit placed into *storage* will yield one unit of good in either periods 1 or 2.

Financial intermediation: The investment technology is operated at a central location, where depositors pool and invest resources collectively in period 0. This arrangement allows them to insure individual liquidity risk and minimize costly liquidation, and we call the arrangement as a *bank*.[3] Each depositor chooses to withdraw in either period 1 or period 2. Depositors are isolated and cannot communicate with each other when they choose the timing of withdrawal.

2.2 Deposit Contract

The bank forms a deposit contract with each depositor, specifying the amount of payments in periods 1 and 2. As in Diamond and Dybvig (1983), the bank pays the pre-fixed amount in period 1 to each withdrawing depositor and pays an even division of the remaining resources in period 2.

[3] The bank is thus a coalition of depositors, but as is standard in the literature, the bank can be interpreted as the outcome of competition for deposits in an economy with free entry.

The bank chooses (c_1, c_2) by solving the following problem:

$$\max_{\{c_1 \geq 0, c_2 \geq 0\}} \pi u(c_1) + (1-\pi)u(c_2)$$

subject to the lifetime resource constraint $\pi c_1 + (1-\pi)\frac{c_2}{R} = 1$. Let (c_1^*, c_2^*) denote the solution to this standard Diamond-Dybvig allocation problem, which satisfies $u'(c_1^*) = Ru'(c_2^*)$. This solution implicitly determine the bank's portfolio choice. Let x denote the fraction of the total endowment placed into investment, and the remaining fraction is placed into storage. Then, we have $x^* = 1 - \pi c_1^* = (1-\pi)c_2^*/R$. As an example, by assuming the CRRA utility function, $u(c) = c^{1-\gamma}/1-\gamma$, with the coefficient $\gamma > 0$, I obtain the following solution.

$$(c_1^*, c_2^*) = \left(\frac{1}{\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}}, \frac{R^{\frac{1}{\gamma}}}{\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}} \right).$$

When the bank makes repayments in period 1, it keeps paying c_1^*. If there is no bank run, the bank will pay c_2^* in period 2. If the bank does not have enough resources to repay c_2^* in period 2 due to a bank run, it will pay an even division of the remaining resource. Note that the crisis is a zero measure event here as in Diamond and Dybvig (1983) and Ennis and Keister (2009). To ensure that the bank is illiquid, we make the same assumption as Ennis and Keister (2009):[4]

$$rR^{\frac{1-\gamma}{\gamma}} < 1. \tag{1}$$

This condition implies that the bank's resources in period 1, including storage and liquidated investments, are not sufficient to pay c_1^* to all depositors.

2.3 Sequential Service

While the setup so far follows the standard Diamond-Dybvig framework, we now introduce sub-periods within period 1 and incorporate randomness in the measure of depositors who withdraw in

[4] In Ennis-Keister, the liquidation value is $1-\tau$ while we use r to denote the liquidation value.

each sub-period.[5] Specifically, period 1 consists of sub-periods indexed by $h = \{0, 1, 2, ..H\}$ where H is sufficiently large. At sub-period $h = 0$, the bank begins repaying depositors in the order they arrive, and the first π measure of withdrawals are processed. If additional depositors request withdrawals, then at each sub-period $h \geq 1$, one of two events occurs: with probability $1 - p$, a measure $\delta > 0$ of depositors withdraws; with probability p, all remaining depositors withdraw. This stochastic process captures the possibility of rapid escalation in withdrawals, as observed in digital bank runs. Within each sub-period, withdrawals are processed on a first come first served basis, and the bank honors the fixed repayment c_1^* specified in the deposit contract. The bank cannot adjust payment amounts across sub-periods (e.g. ATM). Although depositors are randomly assigned to sub-periods, they do not know in advance their position in the queue or the realization of withdrawals in other sub-periods. As in Wallace (1988), each withdrawing depositor consumes immediately and exits the banking location before the next arrives.

2.4 Banking Authority

We now introduce a benevolent banking authority that can suspend the bank's payments during period 1. The authority chooses the timing of suspension ex-post optimally, but lacks commitment power. As a result, the suspension decision is made sequentially, based on the realized history at each sub-period. At each sub-period $h \geq 1$, the authority observes how many depositors have withdrawn since $h - 1$ and chooses whether to suspend the bank's payments.[6] If it suspends, no further withdrawals occur in period 1, and the bank resumes operation only in period 2. In such a case, the remaining impatient depositors do not receive repayment, while the remaining patient depositors can return and are repaid in period 2. If payments are not suspended at sub-period h, the bank must process the next withdrawals, which will be either a measure δ or the entire remaining

[5] This modification does not fundamentally change the sequential service constraint in the spirit of Wallace (1988). What matters for our paper is not how banks repay depositors, but how frequently the policymaker has the opportunity to intervene. The use of sub-periods makes it easier to express the authority's sequential decision.

[6] As explained later, there are always at least π withdrawals because impatient depositors withdraw in period 1. Thus, at the end of $h = 0$, the authority cannot determine whether a run is underway. Since we assume that a bank run is unexpected at the contracting stage in period 0, the authority is assumed not to intervene at the end of period $h = 0$.

mass of depositors, depending on the realization. The authority can revisit the suspension decision at the next sub-period $h + 1$. The key idea of this setup is that the authority faces uncertainty about whether it will have another opportunity to intervene. With probability $1 - p$, a measure δ withdraws and the authority has another opportunity to suspend payments. With probability p, all remaining depositors withdraw before the authority's next opportunity to suspend payments. Our interest lies in how this ex-post incentive interacts with depositors' ex-ante choices. Since depositors anticipate future policy responses, their expectations about the timing of intervention influence whether they choose to withdraw early. In equilibrium, these expectations are correct.

2.5 Timeline

The sequence of events unfolds as follows. In period 0, depositors and the bank form a deposit contract that specifies the amount of repayment in future periods. In period 1, each depositor privately learns her type and chooses whether to withdraw in period 1 or period 2. Depositors are isolated and cannot communicate with one another when making this decision. Those who choose to withdraw in period 1 are randomly assigned to a sub-period, and the first π measure of withdrawals takes place in $h = 0$. If no more than π depositors choose to withdraw in period 1, withdrawals stop at $h = 0$. If more than π depositors choose to withdraw in period 1, the withdrawal process continues into sub-periods $h \geq 1$. At each subsequent sub-period $h \geq 1$, depositors withdraw first, and then the banking authority observes the realized measure of withdrawals and decides whether to suspend payments. This decision is made sequentially across sub-periods. The bank continues to make payments until either the authority suspends withdrawals, the bank serves all period 1 depositors, or the bank exhausts its available funds.

3 Equilibrium without Uncertainty ($p = 0$)

We study a simultaneous-move game played by all depositors at the beginning of period 1. The banking authority, on the other hand, is not a strategic player but rather is the entity determining the payoff function. Anticipating the authority's decision, depositors calculate expected payoffs and choose to withdraw either in periods 1 or 2. We study Nash equilibrium in this environment. In the analysis below, we focus on the incentives of patient depositors because impatient depositors always withdraw in period 1. We define a positive measure of withdrawals by patient depositors as a *bank run*, and when a bank run constitutes an equilibrium, the bank is said to be *fragile*.

This game always has a no-run equilibrium in which all patient depositors withdraw in period 2. In such a case, impatient depositors consume c_1^* and patient depositors consume $c_2^* > c_1^*$. Whether another, bank run equilibrium exists or not depends on the authority's suspension policy. If there is no intervention, the analysis collapses to the original Diamond and Dybvig. Assumption (1) ensures that the bank cannot repay c_1^* to all depositors, and in case of a bank run, withdrawing in period 2 leads to zero repayment. Thus, a bank run equilibrium always exists under Assumption 1.

If instead there is an intervention, whether a bank run equilibrium exists depends on how quickly the authority intervenes. Ennis and Keister (2009) study the same problem, but the withdrawal path is deterministic once the run begins. Although the authority makes sequential decisions, its problem can be solved as a static optimization given the known path of outflows. In contrast, our model introduces uncertainty from the authority's perspective: if it does not suspend in the current sub-period, there is a chance that all remaining depositors withdraw before the next decision point. This distinction captures the risk imposed by rapid and unpredictable outflows in digital bank runs.

3.1 The Ex-Post Efficient Intervention

We begin our analysis with the extreme case, $p = 0$. In this case, the bank repays to a measure δ withdrawals at each sub-period, and there is no uncertainty. When the authority suspends payments at h', the repayment in period 2 will be[7]

$$c_2(\delta h') = \frac{R(x^* - \delta h' \frac{c_1^*}{r})}{(1-\pi)(1-\pi-\delta h')}, \forall h' \geq 1.$$

[7]Following Ennis and Keister (2009), we assume that only the remaining patient depositors share the remaining resources.

To formalize the authority's decision problem under uncertainty, we define the value functions associated with its two choices at each sub-period $h \geq 1$: either suspend withdrawals or allow them to continue. The value of suspension, denoted by $V^S(h)$, is the utility from allocating the bank's remaining assets to the remaining patient depositors in period 2. The value of non-suspension, $V^N(h)$, represents the continuation payoff of proceeding to the next sub-period, with the option to suspend in the future. Since the authority lacks commitment and makes decisions sequentially, its problem takes the form of a dynamic programming problem. At each sub-period $h \geq 1$, the value of non-suspension is given by the Bellman equation:

$$V^N(h) = \delta u(c_1^*) + \max\{V^S(h+1), V^N(h+1)\},$$

reflecting deterministic withdrawals and no uncertainty under $p = 0$. The value of suspension is given by

$$V^S(h) = (1 - \pi - \delta h)(1 - \pi)u(c_2(\delta h)).$$

The authority chooses to suspend if and only if $V^S(h) \geq V^N(h)$. These payoff function satisfies the following property:

Lemma 1 *Suppose $p = 0$. Then, the value functions $V^S(h)$ and $V^N(h)$ are strictly decreasing in h, and they can cross at most once. Let $h^* := \min\{h \in \mathbb{N}_{\geq 1} \mid V^S(h) \geq V^N(h)\}$. Then:*

- *If $V^S(1) \geq V^N(1)$, then $h^* = 1$ and the authority suspends immediately.*
- *If $V^S(1) < V^N(1)$ and the functions cross at some $h \geq 1$, then h^* is the unique sub-period at which $V^S(h) \geq V^N(h)$, and the authority suspends at h^*.*
- *If $V^S(h) < V^N(h)$ for all $h \in \mathbb{N}_{\geq 1}$, then $h^* = \infty$ and the authority never suspends.*

This result guarantees that the authority's suspension policy takes a threshold form: if the authority prefers continuation at some sub-period, it will also prefer continuation at all earlier sub-periods. In particular, Lemma 1 ensures that the threshold h^* is uniquely defined and captures the authority's optimal suspension decision, either as the sub-period at which suspension occurs or as the indication that the authority never intervenes.

Our interest is whether the authority suspends the bank's repayments early enough to prevent a run. Lemma 1 implies that the authority's decision to suspend can be characterized by a threshold: if it does not suspend at some sub-period, it will not suspend at any earlier sub-period. This structure allows us to evaluate the timing of intervention by comparing the value functions at a single point. We define $\hat{\pi}$ as the total measure of early withdrawals at which patient depositors are indifferent between withdrawing early and waiting:

$$c_2(\hat{\pi}) = c_1^*.$$

Since depositors form rational expectations about when the authority will suspend, the threshold $\hat{\pi}$, which equates their payoffs across periods, provides a natural benchmark for fragility. Solving explicitly for $\hat{\pi}$, we have the following result.

Lemma 2 *A patient depositor run on the bank if and only if she expects the cumulative measure of early withdrawals to exceed* $\hat{\pi} \equiv \frac{(1-\pi)\left(R^{\frac{1}{\gamma}}-1\right)+\frac{R}{r}\pi}{\frac{R}{r}-(1-\pi)}$.

Given this threshold, we evaluate whether the authority suspends repayments early enough to prevent a run. Since sub-periods are indexed by integers $h \in \mathbb{N}$, the authority can only intervene at discrete decision points. We therefore define $\bar{h}$ as the first sub-period at which cumulative early withdrawals meet or exceed $\hat{\pi}$. The following result characterizes exactly when a bank run equilibrium exists.

Proposition 1 *A bank run equilibrium exists if and only if $V^N(h) > V^S(h)$ for all $h < \bar{h}$, where $\bar{h} := \min\{h \in \mathbb{N} \mid \pi + \delta h \geq \hat{\pi}\}$.*

Since $\hat{\pi}$ is generally a real number, it may not align with any sub-period at which the authority can act. The threshold $\bar{h}$ therefore identifies the latest point by which intervention must occur to prevent cumulative withdrawals from exceeding $\hat{\pi}$. If the authority waits until $h \geq \bar{h}$, patient depositors anticipate that they will be repaid less than c_1^* in period 2 and thus choose to withdraw early. As a result, a run equilibrium arises. To prevent a run, the authority must suspend strictly before $\bar{h}$.

3.2 A Necessary Condition

We now turn to studying the condition under which a bank run equilibrium does not exist. In particular, we focus on a necessary condition for the bank to be run-proof by evaluating the value functions at $(\hat{\pi} - \pi)/\delta$. Because $(\hat{\pi} - \pi)/\delta$ may not be an integer, we evaluate them by extending $V^N(h)$ and $V^S(h)$ to continuous arguments, denoted $\tilde{V}^N(h)$ and $\tilde{V}^S(h)$.[8] If $\tilde{V}^N((\hat{\pi} - \pi)/\delta) < \tilde{V}^S(\hat{\pi} - \pi)/\delta)$, then the authority suspends repayment at or before $\hat{\pi}$.[9] However, this comparison does not establish a sufficient condition for avoiding a run, because $(\hat{\pi} - \pi)/\delta)$ may not coincide with an actual decision node. Even if suspension is optimal at $\hat{\pi}/\delta$, the authority may still have preferred continuation at the most recent integer-valued sub-period. Thus, the inequality $\tilde{V}^N(\hat{\pi} - \pi)/\delta) < \tilde{V}^S(\hat{\pi} - \pi)/\delta)$ is a necessary, but not sufficient, condition for eliminating a bank run equilibrium.

Proposition 2 *A necessary condition for the bank to be run-proof is that $\tilde{V}^N(\hat{\pi} - \pi)/\delta) < \tilde{V}^S(\hat{\pi} - \pi)/\delta)$, equivalently:*

$$\delta - (1 - \hat{\pi})(1 - \pi) + \left(\frac{R}{r} \frac{r(1 - \pi)R^{\frac{1-\gamma}{\gamma}} - (\hat{\pi} + \delta - \pi)}{\pi + (1 - \pi)R^{\frac{1-\gamma}{\gamma}}} \right)^{1-\gamma} < 0.$$

The expression is fully characterized by parameters. Under CRRA preferences, it can be used to derive comparative statics that help illuminate the economic determinants of fragility. Specifically,

[8] Formally, $V^N(h)$ and $V^S(h)$ are defined on integers $h \in \mathbb{N}$, while $\hat{\pi}/\delta$ may not be. To evaluate the authority's incentives at the threshold $\hat{\pi}$, we define continuous extensions $\tilde{V}^N(h)$ and $\tilde{V}^S(h)$ over $h \in [1, \infty)$ by linear interpolation. Both functions inherit monotonicity: $V^S(h)$ and $V^N(h)$ are strictly decreasing (See the proof of Lemma 1), and so are their continuous extensions.

[9] Although $(\hat{\pi}-\pi)/\delta)$ and $(\hat{\pi}-\pi)/\delta)+1$ may not be a decision point for the authority, we evaluate the value functions at this index using their continuous extensions. Since the authority's value functions are monotonic, evaluating them at this threshold allows us to study behavior at all earlier sub-periods.

when $\gamma > 1$, we obtain the following results.[10]

Corollary 1 *Suppose the CRRA coefficient $\gamma > 1$. The inequality is less likely to hold as either: (i) the measure δ of interim withdrawals increases, (ii) the fraction π of impatient depositors increases, or (iii) the liquidation value r decreases.*

Since these make the necessary condition for the bank to be run-proof harder to hold, we interpret this result as suggesting that the bank becomes more fragile. A higher δ reduces the frequency of intervention opportunities, making it harder for the authority to act before cumulative withdrawals exceed $\hat{\pi}$. The authority is more likely to miss the appropriate timing and instead delay its intervention. A higher π increases the authority's incentive to continue repayments, as the next depositor in line is more likely to be impatient. Both effects reduce the authority's willingness or ability to intervene early, making the bank more fragile. A higher r, by contrast, makes investment more liquid, allowing the bank to meet early withdrawals with less resource loss. This gives the authority more room to delay intervention without reducing c_2, making the bank less fragile.

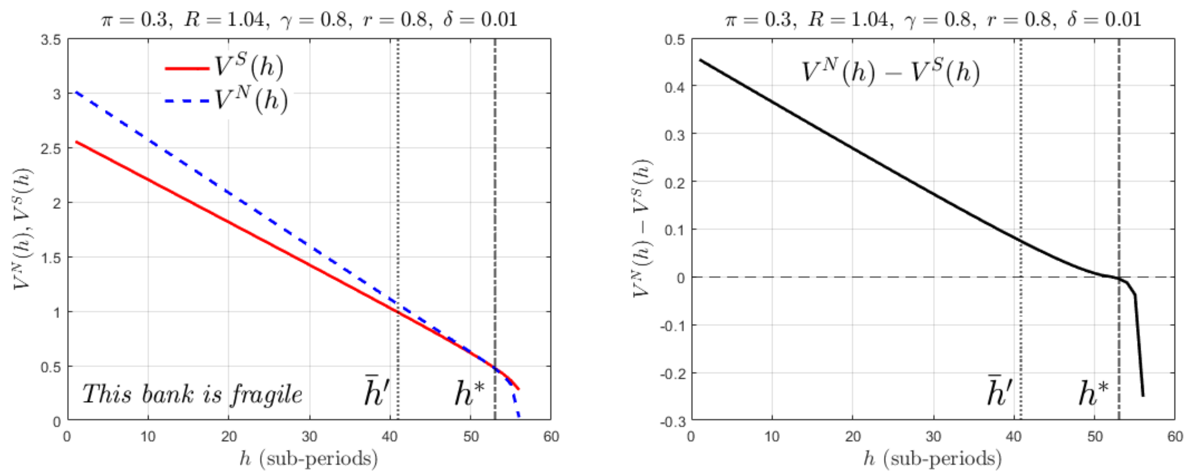
3.3 Numerical Examples

We further study Proposition 1, and we analyze it numerically in this section. To understand how the value functions evolve over h, we illustrate $V^N(h)$ and $V^S(h)$ in Figure 1. As Lemma 1 suggests, both value functions are monotonically decreasing in h and cross only once. The right panel depicts the difference $\left\{V^N(h) - V^S(h)\right\}$ to highlight the position of h^*. In this example, the authority chooses not to suspend until h^*, which exceeds $\bar{h}$. Depositors thus expect $c_2(\delta\bar{h}) < c_1^*$ if all others withdraw in period 1, giving rise to a bank run equilibrium.

[10]When $\gamma > 1$, to ensure $u(0) = 0$ while preserving the CRRA structure, we follow Ennis and Keister (2009) and adopt the shifted utility function

$$u(c) = \frac{(c+b)^{1-\gamma} - b^{1-\gamma}}{1-\gamma},$$

where b is a small positive constant.



(a) $V^N(h)$ and $V^S(h)$ over h

(b) The difference $(V^N(h) - V^S(h))$

Figure 1: A late suspension: $\bar{h} < h^*$

Note: The left panel shows the value functions over h, and the right panel shows the difference between them, $V^N(h) - V^S(h)$. The parameter values used in this example are $(\pi, R, \gamma, r, \delta) = (0.3, 1.04, 0.8, 0.8, 0.01)$. In this example, we illustrate $\bar{h}' = (\hat{\pi} - \pi)/\delta$, and $\bar{h}$ is the smallest integer that satisfies $h \geq \bar{h}'$. The outcome is $(h^*, \bar{h}', \bar{h}) = (53, 40.86, 41)$.

We also investigate the effect of δ on fragility. In Corollary 1, we established that an increase in δ makes the necessary condition for the bank to be run-proof harder to satisfy. We interpreted this as indicating that a larger δ makes the bank more fragile. Here, we numerically study the necessary and sufficient condition in Proposition 1 to examine how δ affects fragility. In Figure 2, we use the value of $\delta = 0.2$ that is larger than the one we used in Figure 1 ($\delta = 0.01$). All other parameter values are the same, and thus the two figures differ only in the value of δ. When we assume a smaller δ, the bank is not fragile because $h^* < \bar{h}$. In contrast, when we assume a larger δ, we find the bank to be fragile. This result is consistent with the comparative statics discussed in the previous subsection. The intuition remains the same: a lower frequency of suspension opportunities makes timely intervention more difficult. By waiting for the next opportunity, the authority can serve more impatient depositors, but that opportunity may not arrive in time. By then, the bank may have already liquidated a substantial portion of its investments, leading to $c_2 < c_1^*$. Even if the authority recognizes this, it may still choose to delay suspension in order to serve additional impatient depositors.[11]

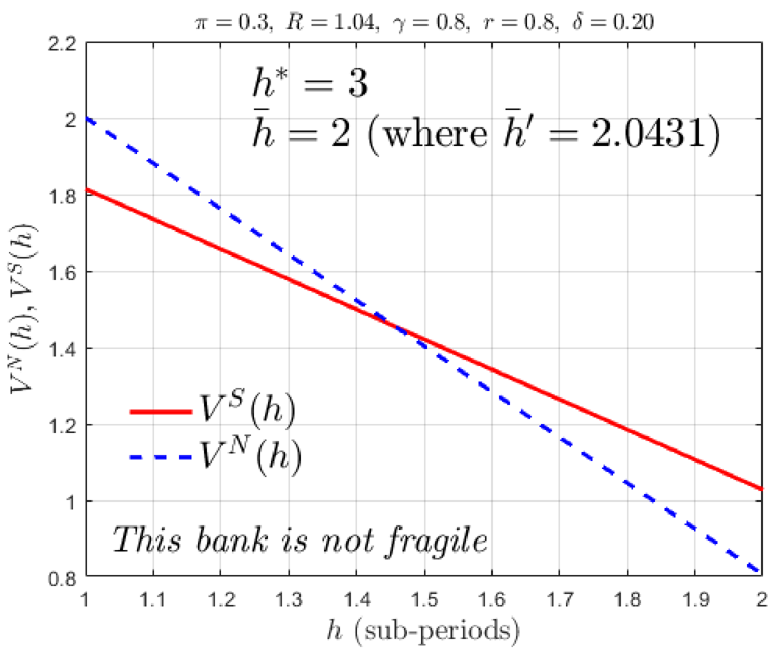


Figure 2: $V^N(h)$ and $V^S(h)$ over h

Note: The parameter values used in this example are $(\pi, R, \gamma, r, \delta) = (0.3, 1.04, 0.8, 0.8, 0.2)$. We again illustrate $\bar{h}' = (\hat{\pi} - \pi)/\delta$, and $\bar{h}$ is the smallest integer that satisfies $h \geq \bar{h}'$. The outcome is $(h^*, \bar{h}', \bar{h}) = (2, 2.04, 3)$.

4 Equilibrium with uncertainty ($p > 0$)

We now extend the analysis to the case where the withdrawal process is stochastic, introducing the possibility that all remaining depositors withdraw at once with probability $p > 0$. This modification captures a key feature of digital bank runs: sudden, unpredictable surges in withdrawal demand

[11]Note that the suspension decision is made after depositors choose when to withdraw. Thus, the authority's action does not influence withdrawal behavior directly. However, depositors form expectations about the authority's behavior and choose whether to run accordingly.

that leave little time for intervention. Unlike the deterministic case ($p = 0$), the authority now faces uncertainty about whether it will have another opportunity to act. This changes the nature of the suspension problem from a deterministic stopping rule to a stochastic one, requiring the authority to weigh the risk of a full-scale run against the benefits of delaying suspension to serve more impatient depositors. We analyze how this uncertainty affects the timing of intervention and the conditions under which a bank run equilibrium emerges.

4.1 Modified Authority's Incentives

We begin the analysis by characterizing the authority's decision problem under uncertainty. At any sub-period $h \geq 1$, suppose the authority has been given opportunities to suspend and has chosen not to intervene in all previous sub-periods. The bank's remaining resources at h are given by $rx^* + (1 - x^*) - h\delta c_1^*$. Given that each depositor receives c_1^*, the measure of depositors the bank can serve additionally at sub-period h is

$$q(h) = \frac{rx^* + (1 - x^*) - h\delta c_1^*}{c_1^*}.$$

Using this expression, the authority's value of continuing without suspension is given by

$$V^N(h) = (1 - p)\left\{\delta u(c_1^*) + \max\{V^S(h + 1), V^N(h + 1)\}\right\} + pq(h)u(c_1^*),$$

where the first term reflects the expected value of reaching the next sub-period, and the second term captures the payoff in the event of a full mass withdrawal by the remaining depositors. In this case, only a measure $q(h)$ of the remaining depositors can be repaid. After serving these additional $q(h)$ depositors, the bank exhausts its resources and goes bankrupt. The value of suspension, $V^S(h)$, remains the same as in the deterministic case.

As in the deterministic case, the value functions $V^S(h)$ and $V^N(h)$ are strictly decreasing in h. The suspension value $V^S(h)$ remains as defined in the previous section and continues to be strictly decreasing by Lemma 1. Under uncertainty ($p > 0$), we now show that the continuation value $V^N(h)$

is also strictly decreasing. These monotonicity properties ensure that the two value functions can cross at most once. As a result, the authority's suspension decision again follows a threshold rule: there exists a unique sub-period h^* such that the authority strictly prefers continuation at all $h < h^*$ and suspends at all $h \geq h^*$. This structure is formalized in the following lemma.

Lemma 3 *Suppose $p > 0$. Then, the value functions $V^S(h)$ and $V^N(h)$ are strictly decreasing in h, and they can cross at most once. Let*

$$h^* := \begin{cases} \min\{h \in \mathbb{N}_{\geq 1} \mid V^S(h) \geq V^N(h)\}, & \textit{if such } h \textit{ exists}, \\ \infty, & \textit{otherwise}. \end{cases}$$

Then h^* is uniquely defined and characterizes the authority's optimal suspension decision.

This result ensures that the authority's suspension policy continues to follow a threshold structure even under uncertainty. The threshold h^* is uniquely determined and describes the authority's behavior: the authority suspends if and only if the realized sub-period is at or beyond h^*. If $h^* = \infty$, the authority never suspends.

Under uncertainty, a patient depositor compares the period 1 payment c_1^* with the expected value of waiting to withdraw in period 2. Even if the authority intends to suspend payments at sub-period h^*, each prior sub-period carries a probability p that all remaining depositors withdraw, removing the chance to intervene. The probability of reaching h^* without such a full withdrawal is $(1-p)^{h^*-1}$. In that case, the depositor receives $c_2(\delta h^*)$ following suspension; otherwise, she receives nothing. The expected payoff from waiting is therefore $(1-p)^{h^*-1} c_2(\delta h^*)$.

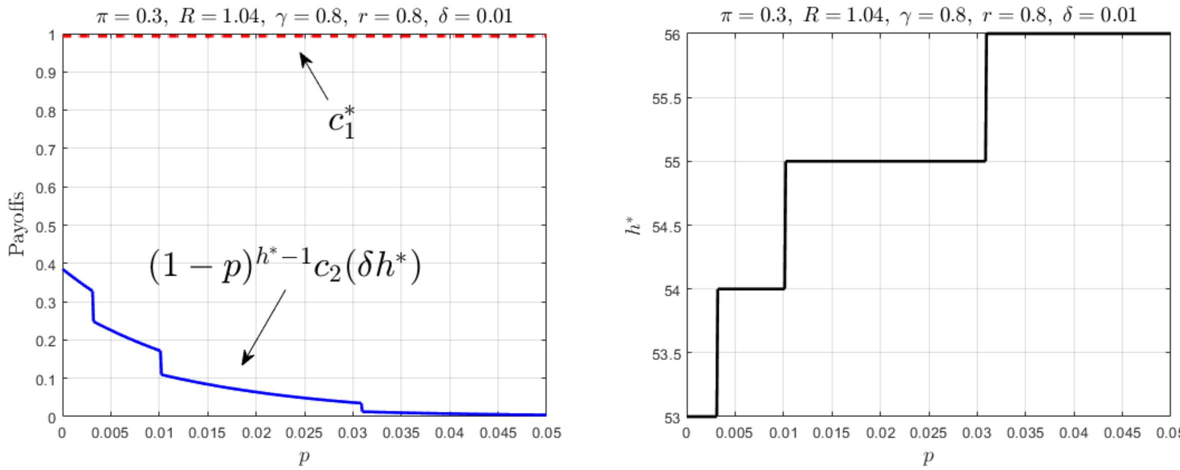
Proposition 3 *A bank run equilibrium exists if and only if*

$$(1-p)^{h^*-1} c_2(\delta h^*) < c_1^*.$$

This condition determines whether a run equilibrium arises in the presence of uncertainty. If the expected payoff from waiting to withdraw in period 2 falls below the period 1 payment, patient depositors strictly prefer to withdraw early, and a run equilibrium arises. If the inequality does not hold, withdrawing in period 2 yields a higher expected payoff, and no run occurs.

4.2 Numerical Examples

To illustrate Proposition 3, we numerically examine how the existence of a run equilibrium depends on the probability p of a sudden full withdrawal. Panel (a) of Figure 3 compares the expected payoff from waiting to withdraw in period 2 with the contractual repayment c_1^* available in period 1. As p increases, the probability of reaching the suspension point h^* without a full withdrawal declines, reducing the expected continuation value. In this example, the expected payoff from waiting falls below c_1^* across all values of p, confirming that a run equilibrium arises. Panel (b) shows how the authority's optimal suspension threshold h^* declines with p, reflecting the growing risk of losing future intervention opportunities.



(a) Payoffs from withdrawing in periods 1 v.s. 2

(b) The optimal stopping point (h^*)

Figure 3: Run incentives and policy response over p

Note: The left panel compares the expected payoff from waiting with c_1^* over p, and the right panel shows the authority's optimal choice of h^*. The parameter values used in this example are $(\pi, R, \gamma, r, \delta) = (0.3, 1.04, 0.8, 0.8, 0.01)$, which is the same as Figure 1. When $p = 0$, the outcome is equivalent to Figure 1. This example shows that the expected payoff of waiting is smaller than c_1^*, which becomes even lower as p increases.

Lastly, we examine how combinations of (p, δ) determine whether the bank is fragile. In this exercise, we consider an empirically plausible set of parameter values, following Eisenbach and Phelan (2022) and Izumi and Li (in press). Specifically, we set $(\pi, R) = (0.1, 1.05)$, where the value of R is based on Gertler and Kiyotaki (2015), implying that illiquid assets yield an annual return of approximately 5%. The value $\pi = 0.1$ reflects the assumption that 10% of depositors withdraw before maturity over a one-year horizon.[12] We also assume that the liquidation value is $r = 0.8$, meaning that 20% of an investment's value is lost upon liquidation. This figure can vary depending on the severity of the crisis. Using these parameter values, we illustrate in Figure 4 the regions of the (p, δ) space in which the bank is fragile or not fragile.

[12] Artavanis, Paravisini, Robles Garcia, Seru, and Tsoutsoura (2022) report that approximately 0.04% of depositors withdraw daily. Extrapolating this figure yields an annual early withdrawal rate of 10.12%, based on Greek data.

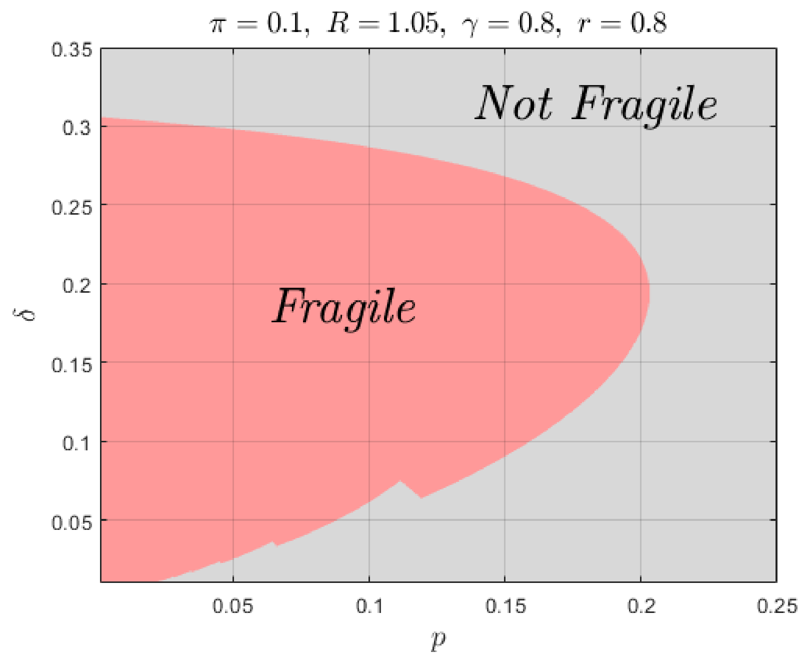


Figure 4: Fragility region over (p, δ)

Note: The parameter values used in this figure are $(\pi, R, \gamma, r) = (0.1, 1.05, 0.8, 0.8)$. The red-colored region indicates that the bank is fragile under the corresponding values of p and δ, while the gray-colored region denotes that no run equilibrium arises. The boundary between these regions may appear jagged due to the integer nature of the suspension threshold h^*, which introduces small discontinuities in the numerical evaluation of fragility.

This numerical exercise illustrates how changes in p and δ affect whether a bank run equilibrium exists. First, increasing p consistently makes the bank less fragile. As p becomes larger, the continuation value $V^N(h)$ declines more quickly because the authority is less likely to have another opportunity to act. This causes the authority to suspend earlier, which helps prevent a run. This pattern is consistent with the mechanism shown in Figure 3. On the other hand, δ has a non-monotonic effect on fragility for some values of p in this example: increasing δ first makes the bank more fragile and then less fragile. The authority would like to stop withdrawals at a precise point that balances help for impatient depositors against preservation of period 2 resources, yet it can intervene only after each block of size δ. It must therefore decide whether to halt one block earlier or one block later than this ideal. When δ is small, stopping later sacrifices little period 2 consumption while allowing another set of impatient depositors to be paid. The authority therefore tends to delay. If the bank is not fragile at very small δ, a slight increase in δ does not change this outcome. But as δ increases further, the authority is more likely to miss the ideal timing and delay too long. The resulting withdrawals push c_2 below c_1^*, and the bank becomes fragile. When δ is large, however, serving an extra block leads to significant resource loss in period 2, making further delay too costly. Anticipating this, the authority suspends earlier, restoring depositor confidence and avoiding a run. As a result, fragility first increases and then decreases with δ, generating the observed non-monotonic pattern over intermediate values of p.

4.3 Discussion

The model provides a framework for understanding how the speed of withdrawals shapes the fragility of the banking system and offers insights into recent episodes such as the 2023 collapse of Silicon Valley Bank (SVB). U.S. Government Accountability Office (2023) reports that, on 9 March 2023, SVB experienced outflows of \$42 billion, about 25 percent of its \$166 billion deposit base, and management warned that another \$100 billion could leave the next morning. In total, depositors attempted to withdraw roughly 85 percent of total deposits within 48 hours. This reflects a situation in which the policymaker may have no further opportunity to intervene before the bank is emptied, a case corresponding to a high value of p in our model.

Digital communication tools, including social networks and messaging platforms, are related to the interpretation of p because they can cause panic to spread instantly. When depositors suddenly react to viral news, the result can resemble a full mass withdrawal within a very short period. Our model captures this scenario as a high p, where the likelihood of losing the next opportunity to intervene becomes large. As shown in Proposition 3 and Figure 3, when p is high, the continuation value falls quickly and the authority chooses to suspend early. Fragility is reduced not because depositors are less aggressive, but because the authority reacts more decisively in response to the threat of being overtaken by the speed of panic. This result suggests that the risk of sudden intensified withdrawals may not increase fragility, because it incentivizes the authority to respond more quickly before further losses accumulate.

The parameter δ can be interpreted in light of institutional constraints and the limits of real time decision making. In the model, δ governs the pace of withdrawals within each sub-period, but more broadly, it can capture how quickly outflows accumulate between opportunities to act. In practice, supervisors may not be able to meet continuously or process incoming data without delay. Procedures, coordination, or legal steps may slow the response. For example, liquidity conditions might be updated every few hours, while withdrawals take place continuously. As shown in Figure 4, a higher δ makes it harder to time the suspension precisely. When the authority waits too long, period 2 repayment may become too low, raising fragility. But if δ becomes very large, further delay would cause losses to rise sharply. In this case, the authority may choose to suspend earlier, which decreases fragility. The result highlights that having more frequent opportunities to intervene does not always reduce fragility. If the authority delays action because it expects more opportunities, the outcome can be worse.

5 Conclusion

This paper studies how the timing and uncertainty of policy intervention shape the fragility of banks facing sequential withdrawals. Building on Ennis and Keister (2009), we introduce stochastic withdrawal process and model the policymaker's suspension decision as a dynamic problem under uncertainty. In contrast to settings where the path of outflows is deterministic, our framework highlights that the authority may lose the opportunity to act altogether, and this risk changes the nature of optimal policy.

Our key consideration is that the speed of withdrawals affects fragility by shaping the timing of intervention. A faster and less predictable outflow process can, in some cases, reduce fragility because it prompts the authority to act earlier. On the other hand, when intervention opportunities are more frequent, the authority may choose to wait too long, allowing excessive withdrawals that reduce future repayments. We have studied the conditions under which a run equilibrium arises by characterizing the authority's incentive.

A higher p makes the authority more likely to suspend early, which reduces fragility. The effect of δ is not monotonic: as δ increases, the bank may first become more fragile, but further increases in δ can make the authority act more cautiously and suspend sooner, reducing fragility. The framework provides insight into recent episodes such as the collapse of Silicon Valley Bank, where fast-moving digital panic left little room for policy response.

These findings suggest that the risk of faster withdrawals does not necessarily increase fragility. Instead, it can lead to earlier intervention, reducing the likelihood of a run. Improving the frequency or timeliness of supervisory oversight may help, as long as doing so does not lead the authority to delay suspension in an attempt to accommodate more depositors.

References

Artavanis, N., Paravisini, D., Robles Garcia, C., Seru, A., & Tsoutsoura, M. (2022). *One size doesn't fit all: Heterogeneous depositor compensation during periods of uncertainty* (Working Paper No. 30369). National Bureau of Economic Research.

Cookson, A., Fox, C., Gil-Bazo, J., Imbet, J., & Schiller, C. (2023). Social media as a bank run catalyst. *Federal Deposit Insurance Corporation*, *23-35*, 1-67.

Diamond, D. W., & Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of political economy*, *91*(3), 401–419.

Eisenbach, T. M., & Phelan, G. (2022, July). Cournot fire sales. *American Economic Journal: Macroeconomics*, *14*(3), 508–42.

Ennis, H. M., & Keister, T. (2009). Bank runs and institutions: The perils of intervention. *American Economic Review*, *99*(4), 1588–1607.

Gertler, M., & Kiyotaki, N. (2015, July). Banking, liquidity, and bank runs in an infinite horizon economy. *American Economic Review*, *105*(7), 2011–43.

Izumi, R., & LI, Y. (2024). Rapid Bank Runs and Delayed Policy Responses. *Wesleyan Economics Working Papers*(2024-006).

Izumi, R., & Li, Y. (in press). Financial stability with fire sale externalities. *Journal of Money, Credit and Banking*.

Philipp J. König, P. M., & Pothier, D. (2024). Optimal timing of policy interventions in troubled banks. *Social Science Research Network*.

Rose, J. (2023). Understanding the speed and size of bank runs in historical comparison. *Economic Synopses,*, *12*, 1–5.

Shy, O. (2025). Faster bank runs. *European Economic Review*, *172*, 104915.

U.S. Government Accountability Office. (2023). *Preliminary review of federal agencies' supervision and response to the march 2023 bank failures.* Retrieved from `https://www.gao.gov/assets/gao-23-106736.pdf` (GAO-23-106736)

Wallace, N. (1988). Another attempt to explain an illiquid banking system: The diamond and dybvig model with sequential service taken seriously. *Federal Reserve Bank of Minneapolis*, *12*(4), 1-16.

A Proofs of Results

Lemma 1. We first show that both $V^S(h)$ and $V^N(h)$ are strictly decreasing in h.

(i) Monotonicity of $V^S(h)$: Recall the suspension value is

$$V^S(h) = (1 - \pi - \delta h)(1 - \pi)u(c_2(\delta h)),$$

where

$$c_2(\delta h) = \frac{R(x^* - \delta h \frac{c_1^*}{r})}{(1 - \pi)(1 - \pi - \delta h)}.$$

As h increases, the remaining resources $x^* - \delta h \frac{c_1^*}{r}$ decrease, and the denominator $(1 - \pi - \delta h)$ also decreases. This makes $c_2(\delta h)$ strictly decreasing in h, and since u is strictly increasing and strictly concave, $u(c_2(\delta h))$ is strictly decreasing. Furthermore, the coefficient $(1 - \pi - \delta h)(1 - \pi)$ is linear and strictly decreasing in h. Therefore, $V^S(h)$ is strictly decreasing in h.

(ii) Monotonicity of $V^N(h)$: The continuation value is

$$V^N(h) = \delta u(c_1^*) + \max\{V^S(h + 1), V^N(h + 1)\}.$$

By the induction hypothesis, both $V^S(h + 1)$ and $V^N(h + 1)$ are strictly decreasing in h, so their maximum is strictly decreasing. The term $\delta u(c_1^*)$ is constant, so $V^N(h)$ is also strictly decreasing in h.

(iii) Single-crossing structure: Define the difference function:

$$D(h) := V^S(h) - V^N(h).$$

Since both functions are strictly decreasing, the difference $D(h)$ is not necessarily monotonic, but the sign of $D(h)$ can change at most once. That is, there can exist at most one $h^* \in \mathbb{N}_{\geq 1}$ such that: $V^S(h) < V^N(h)$ for all $h < h^*$, and $V^S(h) \geq V^N(h)$ for all $h \geq h^*$.

This is because if $V^S(h) \geq V^N(h)$ ever holds, and both functions are strictly decreasing, the inequality will continue to hold at all later h. Therefore, the inequality can become true at most once.

(iv) Definition of h^* and optimal policy: We define:

$$h^* := \min\{h \in \mathbb{N}_{\geq 1} \mid V^S(h) \geq V^N(h)\},$$

if the set is nonempty. If the set is empty (i.e., $V^S(h) < V^N(h)$ for all h), we define $h^* := \infty$. This covers all cases. The authority's optimal policy is to: suspend at h^* if $h^* < \infty$ or never suspend if $h^* = \infty$. ■

Lemma 2. A patient depositor is indifferent between withdrawing early and waiting if and only if $c_2(\hat{\pi}) = c_1^*$, where $\hat{\pi}$ is the total measure of early withdrawals that trigger the suspension. Under the model's repayment rule, this implies

$$\frac{R(x^* - (\hat{\pi} - \pi)c_1^*/r)}{(1-\pi)(1-\hat{\pi})} = c_1^*.$$

Substituting $x^* = 1 - \pi c_1^*$ and rearranging, we obtain

$$\hat{\pi} = \frac{(1-\pi)\big(R^{\frac{1}{\gamma}} - 1\big) + \frac{R}{r}\pi}{\frac{R}{r} - (1-\pi)}.$$

■

Proposition 1. We prove the two directions separately.

Sufficiency: Suppose $V^N(h) > V^S(h)$ for all $h < \bar{h}$, where $\bar{h} := \min\{h \in \mathbb{N} \mid \delta h \geq \hat{\pi}\}$. By Lemma 1, the authority's suspension decision follows a threshold rule: there exists a cutoff h^* such that the authority continues for all $h < h^*$ and suspends for all $h \geq h^*$. Since $V^N(h) > V^S(h)$ for all $h < \bar{h}$, we have $h^* \geq \bar{h}$. Hence, cumulative withdrawals reach at least $\hat{\pi}$ before the authority intervenes. By definition of $\hat{\pi}$, we have $c_2(\hat{\pi}) = c_1^*$. Because $c_2(\cdot)$ is strictly decreasing, it follows that $c_2(\delta h^*) < c_1^*$ for $h^* > \bar{h}$, and $c_2(\delta h^*) \leq c_1^*$ for $h^* = \bar{h}$. Therefore, patient depositors anticipate period-2 consumption weakly below c_1^* and are at best indifferent between withdrawing early and waiting. Hence, withdrawing in period 1 becomes a best response, and a run equilibrium exists.

Necessity: Suppose a bank run equilibrium exists. Then patient depositors choose to withdraw early, which implies that they expect $c_2 < c_1^*$ if they wait. From the definition of $\hat{\pi}$, this implies that cumulative withdrawals exceed $\hat{\pi}$ before the authority suspends. Hence, the authority must not suspend at any $h < \bar{h}$, which requires that $V^N(h) > V^S(h)$ for all $h < \bar{h}$. ■

Proposition 2. The condition is derived by evaluating the authority's value functions at $h = (\hat{\pi} - \pi)/\delta$ using continuous extensions. When $\tilde{V}^N\left(\frac{\hat{\pi}}{\delta}\right) < \tilde{V}^S\left((\hat{\pi} - \pi)/\delta\right)$, Lemma 1 implies $\tilde{V}^N\left((\hat{\pi} - \pi)/\delta + 1\right) < \tilde{V}^S\left((\hat{\pi} - \pi)/\delta + 1\right)$. Substituting the expression for $\tilde{V}^S(h)$ and using the identity $c_2(\hat{\pi}) = c_1^*$, we can rewrite the necessary condition as:

$$\delta u(c_1^*) + (1 - \hat{\pi} - \delta)(1 - \pi)u(c_2(\hat{\pi} + \delta)) < (1 - \hat{\pi})(1 - \pi)u(c_1^*).$$

Assuming CRRA utility, $u(c) = c^{1-\gamma}/(1-\gamma)$, we algebraically simplify both sides. This yields the inequality:

$$\delta - (1 - \hat{\pi})(1 - \pi) + \left(\frac{R}{r} \frac{r(1-\pi)R^{\frac{1-\gamma}{\gamma}} - (\hat{\pi} + \delta - \pi)}{\pi + (1-\pi)R^{\frac{1-\gamma}{\gamma}}} \right)^{1-\gamma} < 0,$$

as the necessary condition for the bank to be run-proof. ■

Corollary 1. We consider how the inequality in Proposition 1 responds to changes in δ, π, and r, under the assumption that $\gamma > 1$.

(i) Effect of δ: The left-hand side is increasing in δ because:

- The first term, δ, increases linearly.
- The second term, which includes the expression $\pi r - (\hat{\pi} + \delta)$ in the numerator, decreases in δ. Since $1 - \gamma < 0$, the entire power term increases in δ.

Therefore, the total left-hand side increases, making the inequality harder to satisfy.

(ii) Effect of π: As π increases:

- The first term, $-(1-\hat{\pi})(1-\pi)$, increases since both $\hat{\pi}$ and π increase with π.
- The second term decreases in the inner expression's numerator (since $r(1-\pi)$ decreases and $\hat{\pi}+\delta-\pi$ increases), which causes the entire power term to increase.

Hence, the total expression increases in π, making the inequality harder to satisfy.

(iii) Effect of r: As r increases:

- The first term is unaffected directly, but $\hat{\pi}$ decreases in r, making $(1-\hat{\pi})(1-\pi)$ larger and the first term more negative.
- In the second term, both $\frac{R}{r}$ decreases and the numerator $r(1-\pi)$ increases. These two effects offset each other in the base, but the overall effect is that the base increases.

Since the exponent $1-\gamma<0$, the power term decreases. Both effects reduce the left-hand side, making the inequality easier to satisfy. ■

Lemma 3. We begin by establishing that both $V^S(h)$ and $V^N(h)$ are strictly decreasing in h.

<u>(i) Monotonicity of $V^S(h)$:</u> The suspension value remains the same as in the case with $p=0$:

$$V^S(h)=(1-\pi-\delta h)(1-\pi)u(c_2(\delta h)),$$

where

$$c_2(\delta h)=\frac{R(x^*-\delta h\frac{c_1^*}{r})}{(1-\pi)(1-\pi-\delta h)}.$$

This function is strictly decreasing in h for the same reasons as in Lemma 1: both the numerator and denominator shrink in h, and the declining resource availability leads to a strictly decreasing $c_2(\delta h)$. Since u is strictly increasing and concave, $u(c_2(\delta h))$ is strictly decreasing. Thus, $V^S(h)$ is strictly decreasing in h.

(ii) Monotonicity of $V^N(h)$: With $p > 0$, the continuation value is given by:

$$V^N(h) = (1-p)\left\{\delta u(c_1^*) + \max\{V^S(h+1), V^N(h+1)\}\right\} + p\, q(h)\, u(c_1^*),$$

where

$$q(h) = \frac{rx^* + (1-x^*) - h\delta c_1^*}{c_1^*}$$

is the measure of depositors that the bank can repay at sub-period h in the event of a full withdrawal. This function $q(h)$ is linear and strictly decreasing in h, and thus $q(h)u(c_1^*)$ is strictly decreasing. Also, by the induction hypothesis, both $V^S(h+1)$ and $V^N(h+1)$ are strictly decreasing, so their maximum is strictly decreasing as well. Multiplying by $(1-p)$ and adding the strictly decreasing term $pq(h)u(c_1^*)$ preserves strict monotonicity. Therefore, $V^N(h)$ is strictly decreasing in h.

(iii) Threshold structure and uniqueness of h^*: Define the difference:

$$D(h) := V^S(h) - V^N(h).$$

While $D(h)$ is not necessarily monotonic, we can use the fact that both $V^S(h)$ and $V^N(h)$ are strictly decreasing to establish a single-crossing structure. That is, if $D(h) \geq 0$ for some h, then for all $h' > h$, we must also have $D(h') \geq 0$. Conversely, if $D(h) < 0$, then $D(h') < 0$ for all $h' < h$. This implies that the inequality $V^S(h) \geq V^N(h)$ can become true at most once as h increases.

Therefore, we define:

$$h^* := \begin{cases} \min\{h \in \mathbb{N}_{\geq 1} \mid V^S(h) \geq V^N(h)\}, & \text{if the set is nonempty,} \\ \infty, & \text{otherwise.} \end{cases}$$

This value is uniquely defined. If $h^* < \infty$, then $V^S(h) < V^N(h)$ for all $h < h^*$ and $V^S(h) \geq V^N(h)$ for all $h \geq h^*$. If no such h exists, then $V^S(h) < V^N(h)$ for all h, and the authority never suspends.

This completes the proof. ■